Year 11 Higher Maths KO
Autumn Term 2026

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Year 11 – Maths – Unit A14 – Advanced Quadratic Graphs & Equations

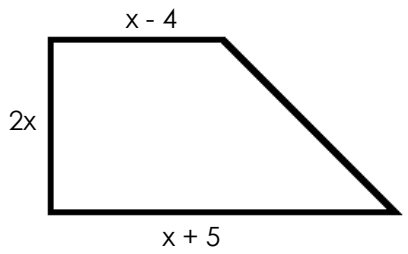
You will need to use your knowledge of solving quadratic equations from Units A11 and A12 to form and solve quadratic equations based on geometric problems. There are two examples of this below, however, these types of problems can be seen in lots of different contexts.

My mathematical journey

What do I need to remember from before?	What will I learn about in this unit?	Where does this lead?
Quadratic equations (A12)	Completing the square when $a \neq 1$	Advanced algebraic manipulation (A17)
Completing the square (A12)	Non-linear simultaneous equations	A-Level Mathematics
Simultaneous equations (A10)	Solving quadratic inequalities	
Solving inequalities (A8)		

Example 1

The area of the trapezium below is 351cm^2



You need to know the formula for the area of a trapezium = $\frac{1}{2}$ (a + b) h

Show that $2x^2 + x - 351 = 0$

$\frac{1}{2}(a + b)h$ ← Substitute in values for a and b and h

$\frac{1}{2}(x - 4 + x + 5)2x$ — Simplify

$\frac{1}{2}(2x + 1)(2x)$ — Expand

$\frac{1}{2}(4x^2 + 2x) = 351$ — ÷ 2

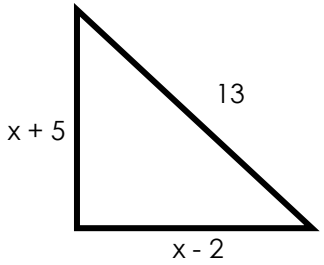
$2x^2 + x = 351$ ← Rearrange

$\underline{\mathbf{2x^2 + x - 351 = 0}}$

This is your answer as it shows the same equation as what is in the question

Example 2

Find the sides of the right angled triangle shown in the diagram.



You need to know Pythagoras theorem. $a^2 + b^2 = c^2$

$(x + 5)2 + (x - 2)2 = 132$ — Substitute

$(x^2 + 10x + 25) + (x^2 - 4x + 4) = 169$ ← Expand

$2x^2 + 6x + 29 = 169$ ← Simplify

$2x^2 + 6x - 140 = 0$ ← Rearrange

$x^2 + 3x - 70 = 0$ — Divide all by 2

$(x + 10)(x - 7) = 0$ ← Factorise

This gives

$x = -10$ *or* $x = 7$

Ignore the -10 as you cannot have a negative side length.

$x = 7$ give side lengths of;

$\underline{\mathbf{5, 12 \text{ and } 13}}$

Keyword/ Skill	Definition/tip
Quadratic (expression/ equation)	An expression/equation involving x^2
Factorise	An expression written as a product of it's factors.
Quadratic formula	A formula for solving quadratic equations. The solution of the equation $ax^2 + bx + c = 0$ is given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Completing the square	A method of solving quadratic equations which involves rewriting the equation $x^2 + px + q$ in the form $(x + a)^2 + b$
Solution	A value or values that we can put in place of a variable (such as x) that makes the equation true.
Coefficient	A number used to multiply a variable. Eg, 3x (3 is the coefficient)

Year 11 – Maths – Unit A14 – Advanced Quadratic Graphs & Equations

Solving quadratic equations by Completing the Square

This method can be used to give answers to a specified number of decimal places or to leave answers in surd form.

Example

Solve $x^2 - 6x - 1 = 0$ by completing the square.

Leave your answer in the form $a \pm \sqrt{b}$.

$$x^2 - 6x = (x - 3)2 - 9$$
$$= (x - 3)2 - 9 - 1$$
$$(x - 3)2 - 10 = 0$$

Rearrange

$$(x - 3)2 = 10$$

$$x - 3 = \pm\sqrt{10}$$

Take the square root of both sides

$$x = 3 \pm \sqrt{10}$$

Finding the turning point from completing the square

In the example above, the quadratic $x^2 - 6x - 1 = 0$ can be written in the form $(x - 3)2 - 10 = 0$

From this equation we can identify that the coordinate of the turning point is

$$(3, -10)$$

The general formula is

If $(x + a)^2 + b$ is our quadratic the coordinate of the turning point will be $(-a, b)$

The line of symmetry for this graph will cross through this point.

The y-intercept of a quadratic graph

The y-intercept for a quadratic graph can be found as the constant c in the equation.

For example, the y-intercept for $x^2 - 6x - 1 = 0$ would be -1.

Solving quadratic equations by Completing the Square where the coefficient of $x^2 > 1$

Example

Solve the quadratic equation

$$2x^2 + 10x = -5 - 2x$$

We first need to ensure our quadratic equation is equal to zero.

$$2x^2 + 12x + 5 = 0$$

Next we need to take out a common factor of 2 so we are left with just x^2

$$2\left(x^2 + 6x + \frac{5}{2}\right) = 0$$

Then complete the square with the factor inside the brackets.

$$2\left((x + 3)^2 - 9 - \frac{5}{2}\right) = 0$$

Then simplify

$$2\left((x + 3)^2 - \frac{13}{2}\right) = 0$$

$$2(x + 3)^2 - 13 = 0$$

Finally rearrange and solve for x

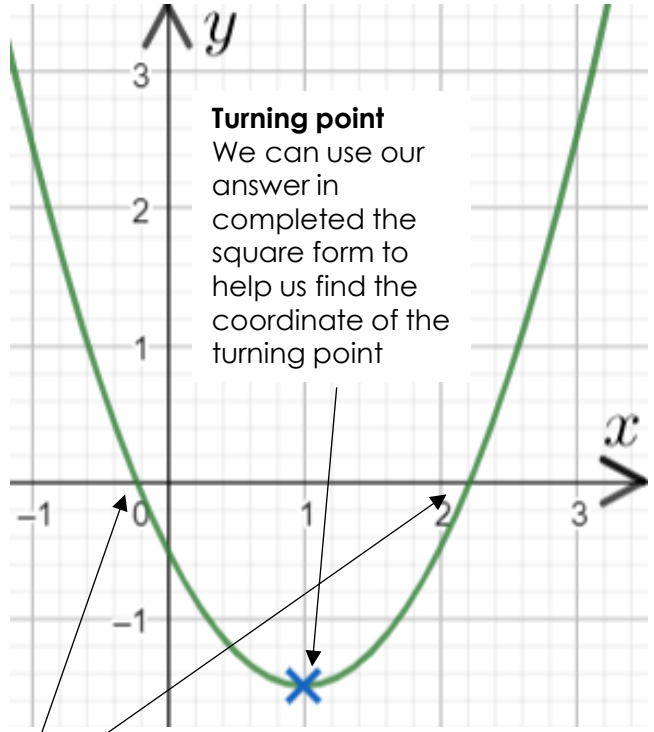
$$2(x + 3)^2 = 13$$

$$(x + 3)^2 = \frac{13}{2}$$

$$x + 3 = \pm\sqrt{\frac{13}{2}}$$

$$x = -3 \pm \sqrt{\frac{13}{2}}$$

Solving a quadratic using a graph



Turning point

We can use our answer in completed the square form to help us find the coordinate of the turning point

Roots – these are the solutions to the quadratic equation. They are the values where the graphs crosses the x-axis.

Year 11 – Maths – Unit A14 – Advanced Quadratic Graphs & Equations

Solving Quadratic Inequalities

The initial process of solving a quadratic inequality is the same as solving a quadratic equation.

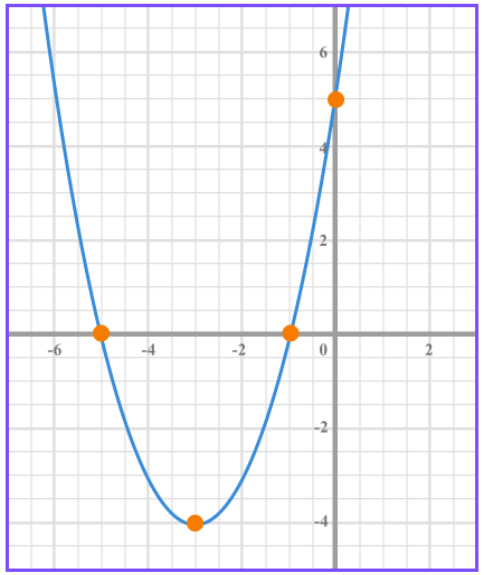
You may do this by factorising, using the formula or completing the square.

The difference is then finding the range of solutions that would satisfy your inequality, rather than just the 1 or 2 solutions that would satisfy an equation.

Example 1

The quadratic equation $x^2 + 6x + 5 = 0$ has two solutions.

This is shown on the graph below where the parabola crosses the x-axis.



We could also solve this by factorising $(x + 1)(x + 5) = 0$
Leading to the two values of $x = -1$ and $x = -5$.

The quadratic inequality $x^2 + 6x + 5 \leq 0$ can be solved by factorising, but instead we need to know the range of x values that give a y value less than zero.

This corresponds to where the graph is below the x axis.

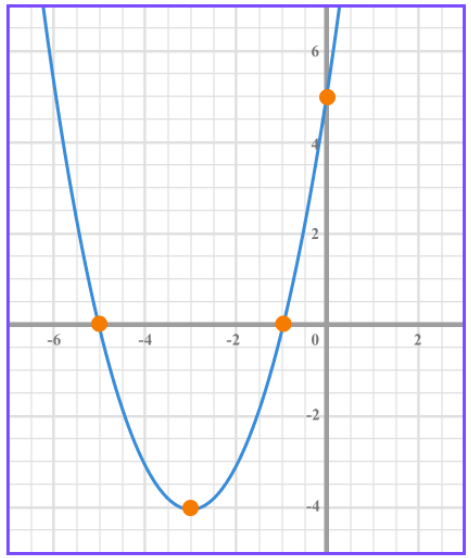
We can see from the graph that the x-values need to be between -1 and -5
Therefore, the solution to this inequality can be written using inequality notation.

$$-5 \leq x \leq -1$$

Example 2

The quadratic inequality $x^2 + 6x + 5 > 0$ is similar to the previous inequality and has a range of solutions.
$x^2 + 6x + 5 > 0$ means that we need to know the x values that give us y values greater than 0.

This corresponds to where the curve is above the x axis.



Again by factorising we can get the critical values for solving the inequality

$$(x + 1)(x + 5) = 0$$

Leading to the two values $x = -1$ and $x = -5$.

The quadratic inequality $x^2 + 6x + 5 \leq 0$ can be solved by factorising, but instead we need to know the range of x values that give a y value less than zero.

We can see the from the graph that the x values need to be greater than -1 and less than -5 to give a y values greater than zero.

Therefore, the solution to this inequality can be written using inequality notation.

$x > -1$ or $x < -5$

Representing inequalities on a number line

You will have seen the following representations of inequalities in A6, and how they are used to represent inequalities on a number line,

Symbol	Circle	Direction of Arrow
<	Open ○	Left
>	Open ○	Right
≤	Closed ●	Left
≥	Closed ●	Right

We use open and/or closed circles to represent inequalities on a number line. A closed circle means that the number **is** included in the represented group of values. An open circle means that the number **is not** included in the represented group of values.

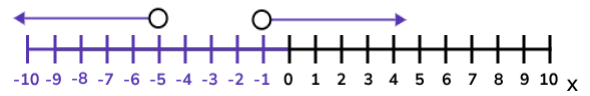
The inequality $x \geq 4$, would be represented like this.



$-2 \leq x < 4$ would look like this.



$x > -1$ or $x < -5$ would look like this.



Year 11 – Maths – Unit A14 – Advanced Quadratic Graphs & Equations

Simultaneous Equations - Substitution

In order to solve simultaneous equations involving non-linear graphs, you will need to be familiar with **substitution method.**

Example 1

Solve the equations: $y = 2x + 3$ and $3x + 4y = 1$ simultaneously.

It is useful to label the equations to help with method.

$y = 2x + 3$ (1)

$3x + 4y = 1$ (2)

You will notice that equation (1) is in the form 'y =' therefore we can use substitution method without rearranging.

We can now substitute what y is equal to from equation (1) into equation (2) to solve for x.

This gives; $3x + 4(2x + 3) = 1$

Now expand and solve;

$$3x + 8x + 12 = 1$$
$$11x + 12 = 1$$
$$11x = -11$$
$$\boldsymbol{x = -1}$$

Substitute $x = -1$ into $y = 2x + 3$ to solve for y.

$$y = -2 + 3$$
$$\boldsymbol{y = 1}$$

CHECK the values are correct for both equations;

$y = 2x + 3$ which gives;

$1 = -2 + 3$

$3x + 4y = 1$ which gives;

$-3 + 4 = 1$

Solutions are correct, **so** $\boldsymbol{x = -1, y = 1}$

Non-Linear Simultaneous Equations

To solve a pair of simultaneous equations when one is linear and the other is non linear (quadratic)

You must always substitute the linear equation into the non-linear equation.

Example 2

Solve these simultaneous equations

$x^2 + y^2 = 5$ (1)

$x + y = 3$ (2)

Rearrange the linear equation (2) to make x or y the subject. Here we have made x the subject.

$$x = 3 - y$$

Now we can **substitute** what x is equal to into equation (1) which gives;

$$(3 - y)2 + y^2 = 5$$

Expand the double brackets and simplify. Ensuring your quadratic is equal to zero so that it can be solved.

$$9 - 6y + y^2 + y^2 = 5$$
$$2y^2 - 6y + 4 = 0$$

We can simplify by dividing the whole equation by 2

$$y^2 - 3y + 2 = 0$$

Factorise to get 2 solutions for y

$$(y - 1)(y - 2) = 0$$
$$\boldsymbol{y = 1} \text{ or } \boldsymbol{y = 2}$$

Substitute both y values into equation (2) to get the corresponding x values.

When $\boldsymbol{y = 1}$, $x + 1 = 3$

$\boldsymbol{x = 2}$

When $\boldsymbol{y = 2}$, $x + 2 = 3$

$\boldsymbol{x = 1}$

You should always give answers as a pair of values in x and y.

$$\boldsymbol{x = 2, y = 1} \text{ OR } \boldsymbol{x = 1, y = 2}$$

Keyword/ Skill	Definition/tip
Simultaneous Equation	A pair of equations with two unknown variables. Both equations need to be solved at the same time (simultaneously)
Substitution	When a letter or expression in an equation, expression or formula is replaced by a number or an expression.
Rearrange	To change the subject of a formula, so that a different variable is the subject of the formula.
Variable	A symbol for a number that we don't know yet. Often this is a letter such as x or y.
Coefficient	The number in front of an unknown quantity (the letter) in an algebraic term.

Exam Tips

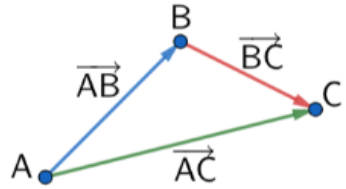
- A non-linear simultaneous question will be worth 4 marks in your exam.
- To get started substitute the linear into the non-linear equation.

Year 11 – Maths – Unit GM10 – Advanced Vectors

Representing Vectors

A vector that goes from point A to point B is denoted as $\overrightarrow{AB}$. We write a vector depending on where it starts and ends.

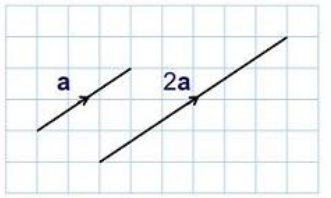
For example, if we add $\overrightarrow{AB}$ to $\overrightarrow{BC}$ the result is a vector that starts at A and ends at C so we write it as $\overrightarrow{AC}$ (it doesn't matter that it goes through B, it matters where it starts and ends)



Parallel Vectors

Vectors are parallel if they are going in the same direction. They do not need to be the same size, they can be **scalar multiples** of the other.

You can prove two vectors are parallel by showing that one is a multiple of the other.

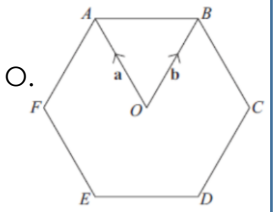


Here **a** and 2**a** are parallel and 2**a** is twice the length of **a**

Solving Geometric Problems

You will have to use vectors to solve geometrical problems.

Example:
ABCDEF is a regular hexagon with centre O.
$\overrightarrow{OA}$ = a $\overrightarrow{OB}$ = b



Write the vector $\overrightarrow{AB}$ in terms of **a** and **b.**

$\overrightarrow{OA}$ = **a** so going in the opposite direction will be $\overrightarrow{AO}$ = -**a**

To get vector $\overrightarrow{AB}$ we can add vectors:

$\overrightarrow{AO}$ and $\overrightarrow{OB}$ → $\overrightarrow{AO} + \overrightarrow{OB}$ = -**a** + **b**

Therefore $\overrightarrow{AB}$ = -**a** + **b**

Solving Geometric Problems with Ratio

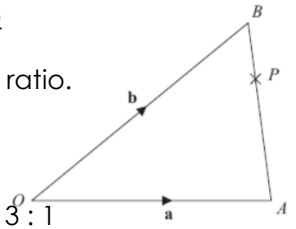
Some vector problems will also involve ratio.

Example:
OAB is a triangle. $\overrightarrow{OA}$ = **a** $\overrightarrow{OB}$ = **b**

a) Find $\overrightarrow{AB}$ in terms of **a** and **b**

P is the point on AB such that AP : PB = 3 : 1

b) Find $\overrightarrow{OP}$ in terms of **a** and **b** (Give your answer in it's simplest form)



For part a we can add the vectors $\overrightarrow{AO}$ and $\overrightarrow{OB}$ together:
$\overrightarrow{AO}$ = -**a** $\overrightarrow{OB}$ = **b** $\overrightarrow{AO} + \overrightarrow{OB}$ = -**a** + **b** $\overrightarrow{AB}$ = -**a** + **b**

For part b, we need to find the vector $\overrightarrow{OP}$. Always think of the path you need to take first (There may be more than one path). The path we need is $\overrightarrow{OA} + \overrightarrow{AP}$
We know $\overrightarrow{OA}$ = **a**

$\overrightarrow{AP}$ is $\frac{3}{4}$ of the path of $\overrightarrow{AB}$ so we need $\frac{3}{4}$ of -**a** + **b** = $-\frac{3}{4}$**a** + $\frac{3}{4}$**b**

Then $\overrightarrow{OA} + \overrightarrow{AP}$ = **a** $-\frac{3}{4}$**a** + $\frac{3}{4}$**b** = $\frac{1}{4}$**a** + $\frac{3}{4}$**b**

Therefore, $\overrightarrow{OP}$ = $\frac{1}{4}$**a** + $\frac{3}{4}$**b**

Exam!

If you need to prove two vectors are parallel then you need to show that one is a multiple of the other.

Example: If you have the vectors **a** + **b** and 2**a** + 2**b**
For the second vector you can factor out 2 so it leaves you with:
a + **b** and 2(**a** + **b**) this shows one is a multiple of the other which proves they are parallel.

Keyword/Skill	Definition/Tips
Vector	A quantity having a direction as well as magnitude. Determines the position of one point in space to another. Vectors can be denoted in as column vectors or like this: 2**a** or $\overrightarrow{AB}$
Direction	Where something is pointing. (Left, right, up, down north south are all directions).
Magnitude	A number that gives the size of something. We represent the magnitude of a vector with $\lvert a \rvert$
Scalar	A single number we multiply a vector by. E.g. We can multiply the vector $\binom{2}{5}$ by the scalar 3 and get $\binom{6}{15}$
Multiple	The result of multiplying a number by an integer.
Parallel	Always the same distance apart and they will never meet.
Collinear	When three or more points lie on a straight line.
Ratio	Shows the relative sizes of two or more values
Column Vector	In a column vector, the top number moves left (-) or right (+) and the bottom number moves up(+) or down(-).

Magnitude of a Vector

We calculate the magnitude of a vector by using Pythagoras' Theorem:

Vector $\boldsymbol{a} = \binom{x}{y}$ $\lvert a \rvert = \sqrt{x^2 + y^2}$

Example: What is the magnitude of vector a $\binom{5}{12}$?

$$\lvert a \rvert = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$$

Other Topics/Units this could appear in:
- Core - Vectors
- Mechanics - Kinematics & Forces and Newton's Law

Year 11 – Maths – Unit NP15 – Indices & Surds

Surds

A surd is an irrational number that is the square root of a positive whole number. The value of the surd cannot be determined exactly, since they have infinite non-recurring decimals. This means they cannot be written as a fraction. Instead, we leave them in surd form so that they are accurate and easier to complete calculations with.

$\sqrt{2} = 1.41421356\ldots$ which never repeats

Rules of Surds

General Rule	Example
$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$	$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	$\sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$
$a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$	$2\sqrt{5} + 7\sqrt{5} = 9\sqrt{5}$
$\sqrt{a} \times \sqrt{a} = a$	$\sqrt{7} \times \sqrt{7} = 7$

Simplifying Surds

We need to use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ to help us with this as it allows us to take out a factor that is a perfect square.

Example - Write $\sqrt{12}$ in the form $a\sqrt{b}$

$\sqrt{12}$ can be written as the product of a rational number and a surd;

$$\sqrt{12} = \sqrt{4 \times 3}$$
$$= \sqrt{4} \times \sqrt{3}$$

Because 4 is a perfect square, this can be simplified to

$$\sqrt{4} \times \sqrt{3} = 2 \times \sqrt{3}$$
$$= 2\sqrt{3}$$

Note that, like in algebra, $2 \times \sqrt{3}$ is written as $2\sqrt{3}$. This is called a mixed surd.

Rationalising the Denominator

This is the process of changing the way a fraction is written so that the denominator contains only rational numbers. But we must not change the value of the fraction.

Example 1

To rationalise the denominator of $\frac{\sqrt{3}}{\sqrt{2}}$ we need to multiply it by a special version of 1, so that we don't change the value of the original fraction.

$$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$
$$= \frac{\sqrt{6}}{\sqrt{4}}$$
$$= \frac{\sqrt{6}}{2}$$

This fraction is equal to 1 but the numerator and denominator are the same as the denominator in the original fraction

We now have a fraction where the denominator is rational, but the value is still equal to the original fraction.

Example 2 - Rationalise the denominator; $\frac{5}{3-\sqrt{2}}$

$$\frac{5}{3-\sqrt{2}} = \frac{5}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$$
$$= \frac{5(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$$
$$= \frac{15+5\sqrt{2}}{9+3\sqrt{2}-3\sqrt{2}-\sqrt{4}}$$
$$= \frac{15+5\sqrt{2}}{9-2}$$
$$= \frac{(15+5\sqrt{2})}{7}$$

This fraction is equal to 1 but this time the surd in the numerator and denominator are the negative of the surd in the original fraction. This is so that when we expand the brackets in the next step, they cancel each other out.

Here we have expanded brackets, the same way you would in algebra. If you need help with this look back at the **Crossover Unit 19 & 20 – Expanding and Simplifying/Factorising** knowledge organiser.

Calculator Skills

In the case that a surds question appears on a calculator paper, or you just want to double check your work, you input surds by using the square root button on your calculator. A calculator will always put your answer in its simplest form.



Keyword/Skill	Definition/Tips
Irrational number	A number that can be written as a decimal but not a fraction.
Surd	The square root of an integer that is irrational. $\sqrt{4} = 2$ (not a surd) $\sqrt{5} = 2.236067\ldots$ (this number is irrational, so $\sqrt{5}$ is a surd)
Perfect square	A number whose square root is an integer. Perfect squares include; 1,4,9,16,25… 3 x 3 = 9
Simplify	To simplify a surd you reduce the number under the square root to the smallest number it can possibly be, by removing a factor that is a perfect square.
Rationalise the Denominator	The process of rewriting a fraction so that the denominator contains only rational numbers.
Factor	A factor is a whole number that divides exactly into another number, they can be written in pairs of numbers that multiply to make that number. Factors of 12 are: 1 and 12 (1 x 12 = 12) 2 and 6 (2 x 6 = 12) 3 and 4 (3 x 4 =12)

Other Topics/Units this could appear in:

- A-level units

Exams!

- It is much more likely for surds to appear on a non-calculator paper so the examiner can see you manipulating surds, so it is crucial in these questions to show every step of your working out.
- If a question asks you to "leave your answer in surd form", this means you answer needs to contain a surd, and you do not change it to a decimal.
  E.g. $\sqrt{7}$ and NOT 2.645…

Year 11 – Maths – Unit NP15 – Indices & Surds

Keyword/Skill	Definition/Tips
Index (Plural-Indices)	A small number placed to the upper-right of a number. It shows the number of times the base number is multiplied by itself.
Base	The number the index is acting upon. 3 is the base and 2 is the index. 3^2
Index Form	Writing numbers in terms of powers E.g. 6 x 6 x 6 x 6 = 6^4 <- This is in index form
Index Laws	A collection of rules we use for simplifying expressions
Notation	A system of symbols used to represent something

Multiplication Law

$\boldsymbol{a^m \times a^n = a^{m+n}}$ Ex1: $5^4 \times 5^9 = 5^{4+9} = 5^{13}$

Ex2: $3a^5 \times 4a^6 = (3 \times 4) \times a^{5+6} = 12a^{11}$

Division Law

$\boldsymbol{a^m \div a^n = a^{m-n}}$ Ex1: $8^{12} \div 8^7 = 8^{12-7} = 8^5$

Ex2: $12a^{18} \div 6a^{10} = (12 \div 6) \times a^{18-10} = 2a^8$

Brackets Law

$\boldsymbol{(a^m)^n = a^{m \times n}}$ Ex1: $(10^2)^4 = 10^{2\times4} = 10^8$

Ex2: $(5a^8)^2 = 5^2 \times a^{8\times2} = 25a^{16}$

Power of Zero

Anything to the power of zero is equal to 1.

$\boldsymbol{a^0 = 1}$ $8^0 = 1$ $129487893^0 = 1$

Other Topics/Units this could appear in:

- Negative & Fractional Index Laws
- Surds including rationalising
- Algebraic Fractions
- Algebraic Proof
- Standard Form
- Expanding & Simplifying
- A-Level – Core – Algebra & Functions

It is only possible to use each of these laws when the base is the same.
Sometimes, we can manipulate the base to ensure this is the case.

Ex: $8^3 \times 4^2 \times 2^5$

8 and 4 can both be written as powers of 2, $8 = 2^3$ and $4 = 2^2$

Therefore, we can rewrite this question with all terms having the same base.

$(2^3)^3 \times (2^2)^2 \times 2^5$ This can now be simplified using the laws from above

$2^9 \times 2^4 \times 2^5 = 2^{9+4+5}$

$= 2^{18}$

Year 11 – Maths – Unit NP15 – Indices & Surds

1. Fractional Indices

If an index is a fraction, the denominator of the fraction acts as a "root". The numerator of the fraction acts as a normal power.

Example 1
Evaluate

$$64^{\frac{2}{3}}$$

In this fractional index the denominator is 3. So the first thing we need to find is the **cube root** of the base number 64.

$$\sqrt[3]{64} = 4$$

Then then numerator is 2 so we need to **square** 4.

$$4^2 = 16$$

So,

$$64^{\frac{2}{3}} = 16$$

Example 2
Evaluate

$$\left(\frac{4}{9}\right)^{\frac{3}{2}}$$

Here we need to make sure we apply the rules to both the numerator and the denominator of the base number.

The denominator of the index is 2 so we need to **square root** the numerator and denominator of the base number.

$$\frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

Then, we need to **cube** the numerator and the denominator of this answer, because the numerator of the index is 3.

$$\frac{2^3}{3^3} = \frac{8}{27}$$

So,

$$\left(\frac{4}{9}\right)^{\frac{3}{2}} = \frac{8}{27}$$

2. Negative Indices

If an index is negative is performs the reciprocal.

Example 1
Evaluate

$$4^{-2}$$

First we need to perform the reciprocal.

$$4^{-2} = \left(\frac{1}{4}\right)^2$$

Now we have the reciprocal, the index becomes positive and we can **square** the numerator and the denominator.

$$\left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

Example 2
Evaluate

$$\left(\frac{2}{3}\right)^{-2}$$

First we need to perform the reciprocal.

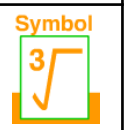
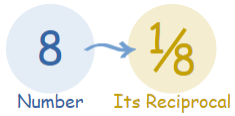
$$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2$$

Now we have the reciprocal the index becomes positive and we can **square** the numerator and the denominator.

$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

Exams!

- Read the question carefully!
- Sometimes it will say "Evaluate" or "What is the value of…", this means it wants the final answer as a single value/number without any indices.
- Sometimes it was say "Simplify" or "Leave your answer in index form", this means you just need to use the laws of indices to simplify the answer, and leave index notation in your answer.

Keyword/Skill	Definition/Tips
Fraction	A part of a group or a whole number. Made up of a numerator and a denominator.
Negative number	A number less than zero. We use – to show a negative number.
Square number	A number that is the result of multiplying an integer by itself.
Cube number	A number that is the result of multiplying an integer by itself 3 times.
Square root	A number, when multiplied by itself gives the original number. SYMBOL: $\sqrt{}$
Cube root	A number, when multiplied by itself three times gives the original number. 
Numerator	The top number in a fraction.
Denominator	The bottom number in a fraction.
Reciprocal	The multiplicative inverse of a number. The product of a number and its reciprocal is always 1. $\frac{1}{2} \times 2 = 1$ $\frac{4}{5} \times \frac{5}{4} = 1$ 

Other Topics/Units this could appear in

- Laws of Indices
- A 'level Units

Year 11 – Maths – Unit NP16 – Numerical & Algebraic Representations of Proportion & Change

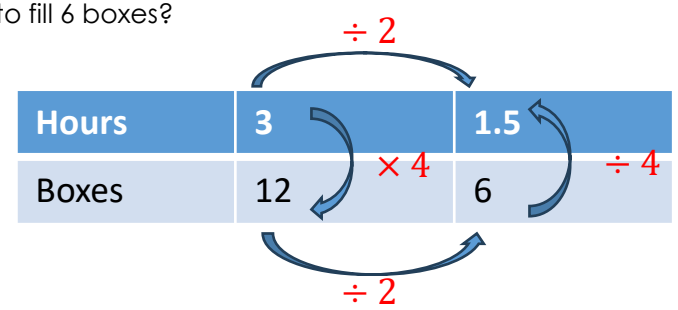
Direct Proportion means as one quantity increases or decreases, so does the other one. Example: The more hours you work, the more you will earn.

If y is directly proportional to x, we represent the relationship as follows $y \propto x$

Then, $y = kx$, where k is the constant of proportionality.

Example 1 – solving numerically using ratio tables

It takes 3 hours to fill 12 boxes with food. How long does it take to fill 6 boxes?



You can use any of the relationships shown in the table above to find the answer, 1.5 hours.

Example 2 – solving algebraically

a is directly proportional to b.
a = 27 when b = 9 Find the value of a when b = 5

$a \propto b$
$a = kb$

Substitute the values for a and b from the question to find k
$27 = k \times 9$
$3 = k$
Rewrite and substitute k into the equation to solve
$a = 3b$
$a = 3 \times 5$
$a = 15$ when $b = 5$

Example 3 – advanced problems

y is directly proportional to the square of x.
When $x = 6, y = 27$. Find y when x = 4.

$y \propto x^2$

$y = kx^2$

$27 = 36k$

$\frac{27}{36} = k$

$0.75 = k$

$y = 0.75x^2$

$y = 0.75 \times 16$

$y = 12$

Example 4 – advanced problems

y is directly proportional to $\sqrt{x}$

x	36	a
y	2	5

Work out the value of a

$y \propto \sqrt{x}$

$y = k\sqrt{x}$

$2 = k \times \sqrt{36}$

$2 = 6k$

$\frac{1}{3} = k$

$y = \frac{1}{3}\sqrt{x}$

$5 = \frac{1}{3}\sqrt{a}$

$15 = \sqrt{a}$

$225 = a$

Keyword/Skill	Definition/Tips
Ratio	Shows the relative sizes of two or more values. E.G. 1 boy and 3 girls would be written as 1:3
Inverse	The opposite or the reverse E.g. the inverse of addition is subtraction.
Proportion	Two ratios or fractions that are equal.
Direct Proportion.	Two quantities change in the same way. When one increases or decrease, so does the other one.
Variable	A symbol for a number we don't know yet, often a letter x or y.
Equation	Says that two things are the equal. (1+1=2).
Linear	A graph that has a straight line.
Substitute	Putting values where the letters are.
Constant of Proportionality	A constant value, written as k, relating to amounts that rise or fall uniformly together.

Other topics/units this may appear in:

- Best Value
- Exchange Rates
- Proportion Recipes
- Linear Graphs

Year 11 – Maths – Unit NP16 – Numerical & Algebraic Representations of Proportion & Change

Inverse Proportion means as one quantity increases, the other decreases.
Example: If one quantity doubles the other is halved. If it takes 4 people 5 hours to dig a hole, it would take 8 people 2.5 hours.
If y is indirectly proportional to x, we represent the relationship as follows $y \propto \frac{1}{x}$

Then, $y = \frac{k}{x}$, where k is the constant of proportionality.

Example 1 – solving numerically using ratio tables

2 cows eat a tonne of food in 6 days. How long would it take 4 cows to eat a tonne of food?

Cows	2	4
Days	6	3

× 2 ÷ 2

$2 \times 6 = 12$ $4 \times 3 = 12$

You can use any of the relationships shown in the table above to find the answer, 3 days.

Example 2 – solving algebraically

y is inversely proportional to x
y = 6 when x = 5. Find the value of y when x = 10

$y \propto \frac{1}{x}$

$y = \frac{k}{x}$

Substitute the values for x and y from the question to find k.

$6 = \frac{k}{5}$

$30 = k$

Rewrite and substitute k into the equation to solve

$y = \frac{30}{x}$

$y = \frac{30}{10}$

$y = 3$

Example 3 – advanced problems

R is inversely proportional to the square root of F. When R = 32, F = 16. Find F when R = 16

$R \propto \frac{1}{\sqrt{F}}$

$R = \frac{k}{\sqrt{F}}$

$32 = \frac{k}{\sqrt{16}}$

$32 = \frac{k}{4}$

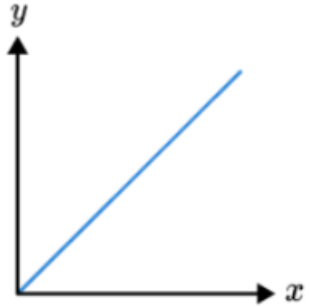
$\boldsymbol{k = 128}$

$16 = \frac{128}{\sqrt{F}}$

$16 = \frac{128}{\sqrt{F}}$

$\sqrt{F} = 8$

$\boldsymbol{F = 82 = 64}$

Directly Proportional Graph	Inversely Proportional Graph
	
Key features: • Algebraic form, $y = kx$ • Always a **straight line graph** • **Positive gradient**, k • Intersects the **origin** (0, 0)	**Key features:** • Algebraic form, $y = \frac{k}{x}$ • Always a **reciprocal graph** • One **smooth curve** • Located in the **first quadrant only** • Does **not touch any axis**

Keyword/ Skill	Definition/Tip
Direct Proportion	Two quantities change in the same way. When one increases or decrease, so does the other one.
Inverse Proportion	As one quantity increases the other decreases.
Equation	The value of two expressions is equal. (Shown by =)
Constant of Proportionality	A constant value, written as k, relating to amounts that rise or fall uniformly together.

Other topics/Units this could appear in:
- A-level
- Pure 1
- Differentiation
- Exponentials and Logarithms
- Pure 2
- Differentiation

Exam Tips
- When you see words, "Inversely Proportional to" replace with "= k ÷" to get you started.
- These questions can range from 2-4 marks.
- You will gain a mark for finding 'k' in each case.

Year 11 – Maths – Unit NP16 – Numerical & Algebraic Representations of Proportion & Change

Interest, Growth and Decay

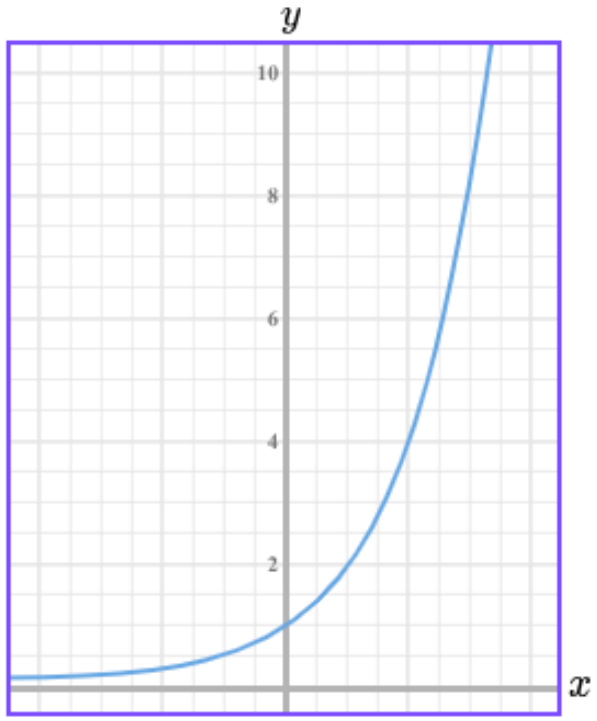
Simple Interest (Growth)	A £200 loan earns 15% **simple** interest over 5 years. How much will be owed at the end of the 5 years? Work out 15% of £200 = £30 £30 x 5 years = £150 £200 + £150 = £350
Compound Interest (Growth)	A £200 loan earns 15% **compound** interest over 5 years. How much will be owed at the end of the 5 years? Here you need to use percentage multipliers. To **increase** by 15% five times (for each of the 5 years) you would multiply by 1.15 five times. A quick way of writing this is by using indices. £200 x 1.15^5 = £402.27
Compound Depreciation (Decay)	A car was brought for £12,000. It depreciates in value by 20% per year. How much will the car be worth after 3 years? Here you need to use percentage multipliers. To **decrease** by 20% three times (for each of the 3 years) you would multiply by 0.8 three times. A quick way of writing this is by using indices. £12,000 x 0.8^3 = £6,144

Exponential Graphs

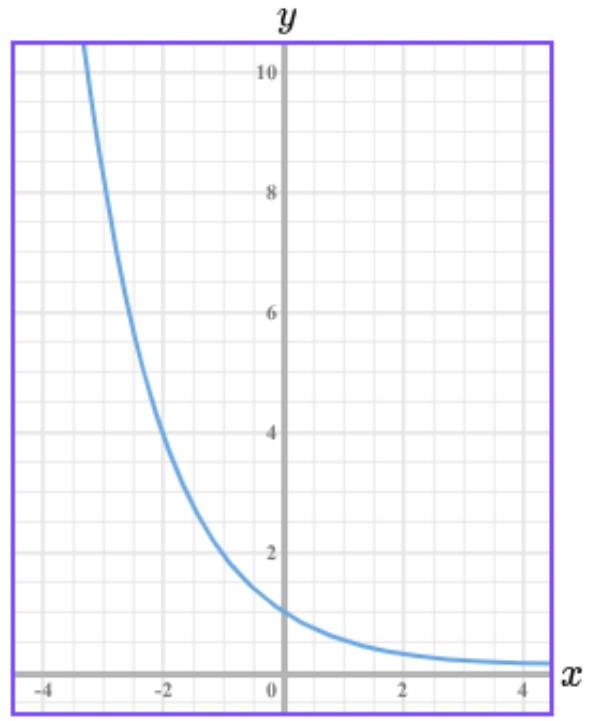
An exponential graph is a representation of an exponential function of the form $y = k^x$.

The graph of an exponential function can represent either **exponential growth** or **exponential decay**.

When k is greater than 1 it is a growth curve.
E.g $y = 2^x$



When k is less than 1 it is a decay curve.
E.g $y = \left(\frac{1}{2}\right)^x$



Sometimes, the phrase 'per annum' is used. This means the same as 'per year'. Annum is the Latin word for year.

Exams!
- A lot of these questions will involve money. Remember money should always be rounded to 2d.p.

Other Topics/Units this could appear in:
- Direct and inverse proportion
- Ratio
- A-level Units

Year 11 – Maths – Unit SP7 – Advanced Probability

Experimental Probability

Experimental probability is the probability of an event happening based on an actual experiment or observation.
To calculate the **experimental probability** of an even, we find the relative frequency of the event.

$$Relative\ frequency = \frac{frequency\ of\ event\ occurring}{total\ number\ of\ trials\ of\ the\ experiment}$$

If we find the relative frequency for all possible outcomes from the experi write the probability distribution.

Number	Frequency	Relative Frequency
1	9	$9 \div 100 = 0.09$
2	16	$16 \div 100 = 0.16$
3	22	$22 \div 100 = 0.22$
4	25	$25 \div 100 = 0.25$
5	21	$21 \div 100 = 0.21$
6	7	$7 \div 100 = 0.07$

In order to calculate an experimental probability distribution:

1. **Draw a table showing the frequency of each outcome in the experiment.**
2. **Determine the total number of trials.**
3. **Write the experimental probability (relative frequency) of the required outcome(s).**

Expected Frequency

Expected frequency is the number of times we would expect an event to occur over a given number of trials that take place during an experiment.

$$\text{Expected frequency} = \text{Probability of event} \times \text{number of trials}$$

In order to find the expected frequency of an event:

1. **Determine the total number of trials of the experiment.**
2. **Determine the probability of the event.**
3. **Multiply the probability of the event by the number of trials of the experiment.**

A game at an amusement arcade has different cash prizes.

The probabilities for winning each cash prize are shown in the table.

Prize	£0	50p	£1	£5
Probability	0.4	0.4	0.15	0.05

It is £1 to play the game and Jonny decides to play the game 40 times.

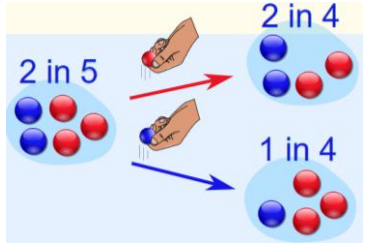
What is the expected value that Jonny will win in total?

Jonny expects to win £0 on $0.4 \times 40 = 16$ occasions.

Keyword/Skill	Definition/Tips
Probability	$\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$
Dependent event	One outcome affects another Example – choosing one red card reduces the chance of choosing another red card
Independent event	One event does not affect the probability of the other.
Mutually exclusive	Two events that cannot happen at the same time, i.e. if you cannot turn left and right at the same time
Conditional	The probability of something happening, given something else has occurred
Sample space	The **set of all possible outcomes** of an experiment.
Theoretical	$\frac{Number\ of\ Favourable\ Outcomes}{Total\ Number\ of\ Possible\ Outcomes}$
Relative Frequency	$\frac{Number\ of\ Successful\ Trials}{Total\ Number\ of\ Trials}$
Experimental Probability	An estimated probability based on the results of an experiment. The more trials are performed, the more reliable the results
Trial	Activity or experiment
Outcome	Result of a trial
Event	One or more outcomes of a trial

Year 11 – Maths – Unit SP7 – Advanced Probability

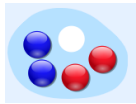
Without Replacement



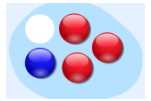
The probability of getting a blue in the first bag is $\frac{2}{5}$

But after taking one out the chances change!

If we got a red marble before, then probability of getting a blue marble from the second bag is $\frac{2}{4} = \frac{1}{2}$



If we got a blue marble before, then probability of getting a blue marble from the second bag is $\frac{1}{4}$

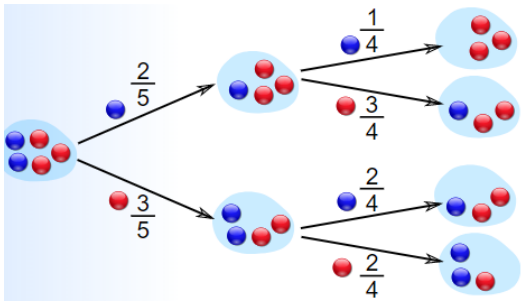


- **With replacement** the events are **independent** (the chances do not change)
- **Without replacement** the events are **dependent** (the chances change)

Tree Diagrams and Conditional Probability

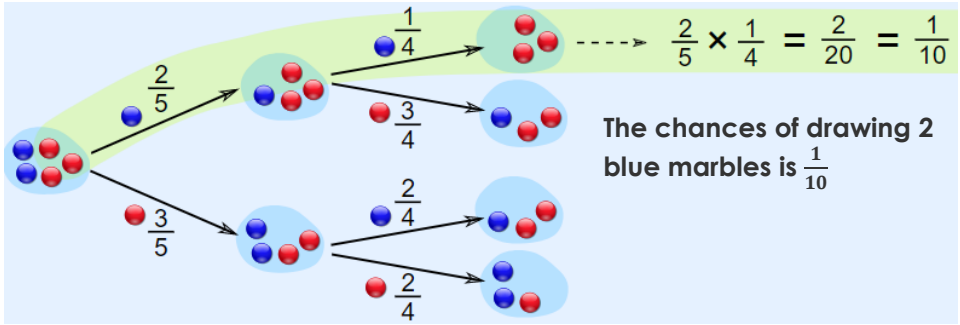
There is a $\frac{2}{5}$ chance of pulling out a Blue marble, and a $\frac{3}{5}$ chance for Red.

We can go one step further and see what happens when we pick a second marble:



Now we can answer questions like **"What are the chances of drawing 2 blue marbles?"**

Answer: it is a $\frac{2}{5}$ **chance** followed by a $\frac{1}{4}$ **chance**



Notation

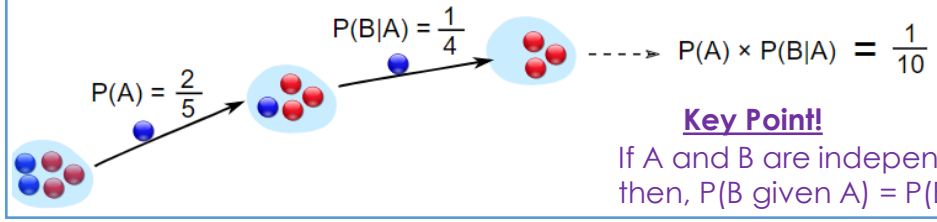
P(A) means "Probability Of Event A"

P(B | A) is also called the "Conditional Probability" of B given A.

P(B | A) means "Event B **given** Event A"

In other words, event A has already happened, now what is the chance of event B?

So the probability of getting **2 blue marbles** is:



"Probability Of" "Given"

$$P(A \text{ and } B) = P(A) \times P(B \mid A)$$

Event A Event B

*Probability of **event A and event B** equals the probability of **event A** times the probability of **event B given event A**"*

Key Point!

If A and B are independent then, P(B given A) = P(B)

Keyword/Skill	Definition/Tips
Probability	$\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$
Dependent event	One outcome affects another Example – choosing one red card reduces the chance of choosing another red card
Independent event	One event does not affect the probability of the other.
Mutually exclusive	Two events that cannot happen at the same time, i.e. if you cannot turn left and right at the same time
Conditional	The probability of something happening, given something else has occurred
Sample space	The **set of all possible outcomes** of an experiment.
Theoretical	$\frac{\textit{Number of Favourable Outcomes}}{\textit{Total Number of Possible Outcomes}}$
Relative Frequency	$\frac{\textit{Number of Successful Trials}}{\textit{Total Number of Trials}}$
Experimental Probability	An estimated probability based on the results of an experiment. The more trials are performed, the more reliable the results
Trial	Activity or experiment
Outcome	Result of a trial
Event	One or more outcomes of a trial

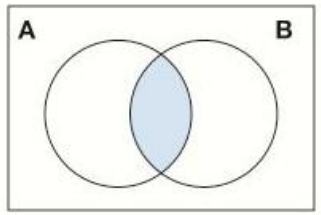
Other Topics/Units this could appear in:

- A Level statistics

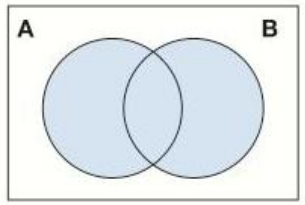
Year 11 – Maths – Unit SP7 – Advanced Probability

Venn Diagram Notation

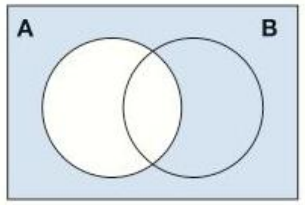
The **intersection** of two sets, A ∩ B consists of the elements common to both sets A *and* B



The **union** of two sets, A U B consists of the elements which appears in at least one of the sets A *or* B



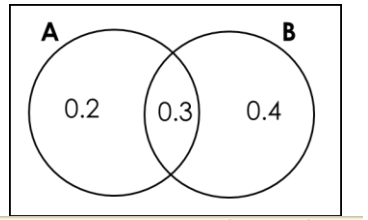
The **complement** of a set A' consistent of the elements which are not in A.



Conditional Probability and Venn Diagrams

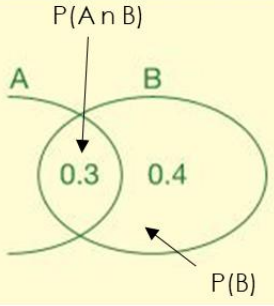
You can use Venn diagrams to calculate conditional probabilities.

The Venn diagram shows the probability attached to two events A and B



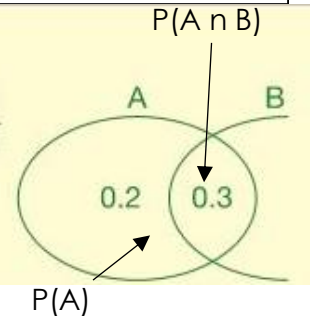
$$P(A \text{ given } B) = \frac{0.3}{0.3 + 0.4} = \frac{3}{7}$$

$$= \frac{P(A \text{ and } B)}{P(B)}$$



$$P(B \text{ given } A) = \frac{0.3}{0.3 + 0.2} = \frac{3}{5}$$

$$= \frac{P(A \text{ and } B)}{P(B)}$$



Conditional Probability and Two Way Table

An insurance company records from 200 accident claims.
The table shows the speed of the car and weather conditions during the accident.

	Wet	Dry	Total
Speeding	11	21	32
Not speeding	77	91	168
Total	88	112	200

A is the event 'it is wet'

B is the event 'a car is speeding'

Are A and B independent?

$P(B) = \frac{32}{200} = 0.16$

$P(B \text{ given } A) = \frac{11}{88} = 0.125$ *(11 out of the 88 cars were speeding when it was wet)*

P(B given A) ≠ P(B), so A and B are not independent

Key Point! *If A and B are independent then, P(B given A) = P(B)*

Keyword/Skill	Definition/Tips
Probability	$\frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$
Dependent event	One outcome affects another Example – choosing one red card reduces the chance of choosing another red card
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Other Topics/Units this could appear in:

- A Level statistics