Year 11 Foundation Maths KO
Autumn Term 2026

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Year 11 – Maths – Unit A9 – Contextual Graphs

My mathematical journey

What do I need to remember from before?

Plotting graphs (A6)

$y = mx + c$ (A6)

Finding gradient (A6)

Ratio tables and direct proportion (NP10, NP11)

What will I learn about in this unit?

Reading, drawing and interpreting graphs used in various contexts

Finding speed

Distance-time and speed-time graphs

Where does this lead?

Compound units: speed, density, pressure and more (NP13)

Gradient of and area under of non-linear graphs (A16)

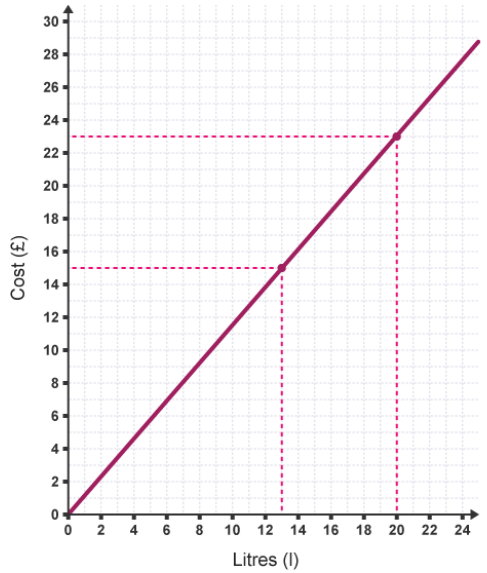
Real Life Graphs

All real-life graphs can be used to estimate or read-off values. The actual meaning of the values will depend on the labels and units shown on each axis.

Sometimes:

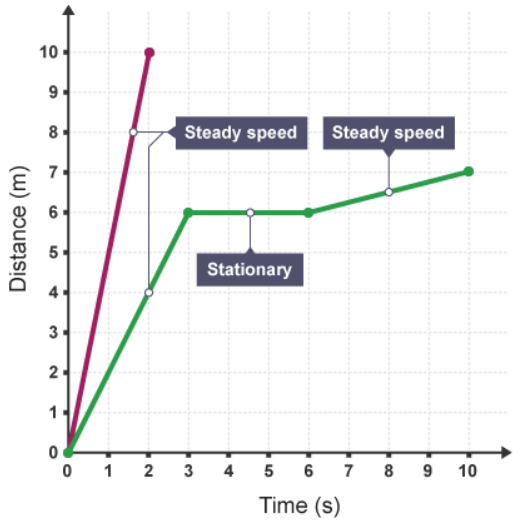
- **the gradient of the line or curve has a particular meaning.**
- **The y-intercept (where the graph crosses the vertical axis) has a particular meaning**
- **The area has a particular meaning**

This graph shows the cost of petrol. It shows that 20 litres will cost £23 or £15 will buy 13 litres.



Distance – Time Graphs

A horizontal line on a **distance-time graph** shows that the object is **stationary** (not moving because the distance does not change)
A sloping line on a distance-time graph shows that the object is moving.
The higher the gradient of a line (how steep it is) the faster the object is moving.



Keyword/Skill	Definition/Tips
Linear	Relating to a line; in a straight direction.
Graph	A drawing or a diagram to record information.
Distance	The length between two points or objects.
Time	Continuum of past to present to future.
Speed	How fast something is moving, how far it travels in a single unit of time. Eg miles per hour, metres per second
Quadrant	Any quarter divided by an x and y axis.
Gradient	How steep a line is.
Intercept	To cross over one another or overlap.
Function	A mathematical relationship between two values.
Convert	To change a value or expression from one form to another

Year 11 – Maths – Unit A9 – Contextual Graphs

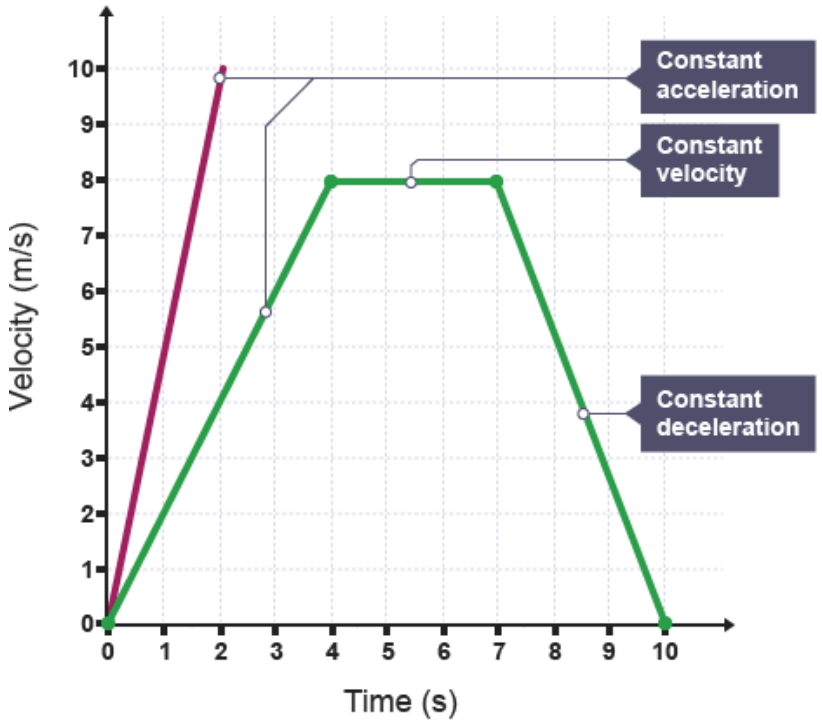
Speed – Time Graphs

A velocity-time graph shows the speed and direction an object travels over a specific period of time.

Velocity-time graphs are also called speed-time graphs.

The vertical axis of a velocity-time graph is the velocity of the object.

The horizontal axis is the time from the start.



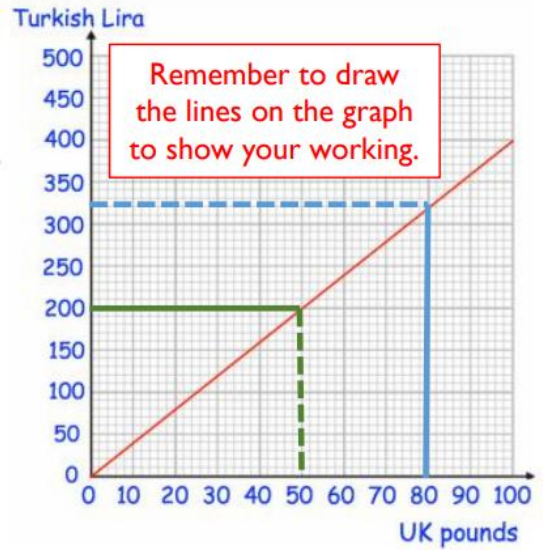
Conversion Graphs

Example 1

Change £80 into Turkish Lira

1) Start at 80 on the x-axes as this for pounds and go up vertically until you reach the line

2) From the line, read horizontally until you get to the y-axis showing Lira. **£80 is approximately 325 Turkish Lira.**



Example 2

Change 600 Turkish Lira to pounds

As this value is not shown by the graph, we have to use a **factor** of 600 to help. So we will use 200 Turkish Lira

1) Start at 200 on the y-axes and go across horizontally until you reach the line. From the line, read vertically until you get to the x-axes.

2) 200 Lira = £50

x3

Turkish Lira	UK Pounds
200	50
600	150

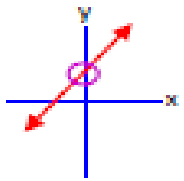
x3

Keyword/Skill	Definition/Tips
Linear	Relating to a line; in a straight direction.
Graph	A drawing or a diagram to record information.
Distance	The length between two points or objects.
Time	Continuum of past to present to future.
Coordinate	Shown as pairs of letters and/or numbers to show position on a coordinate plane or map.
Quadrant	Any quarter divided by an x and y axis.
Gradient	How steep a line is.
Intercept	To cross over one another or overlap.
Function	A mathematical relationship between two values.
Solution	Solving a problem.
Parallel	Always the same distance apart and never touching.
Convert	To change a value or expression from one form to another

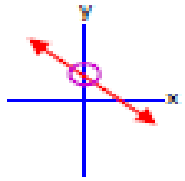
Year 11 – Maths – Unit A10 – Advanced Linear Graphs & Equations

$y = mx + c$

Gradient When gradient is positive.

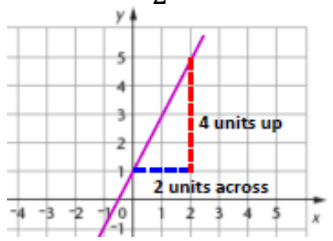


y-intercept When gradient is negative



How to find the gradient of a line.

$$gradient = \frac{change\ in\ y}{change\ in\ x} = \frac{4}{2} = 2$$



My mathematical journey

What do I need to remember from before?

- Solving linear equations (A4)
- Rearranging formulae (A5)
- $y = mx + c$ (A6)
- Linear inequalities (A8)

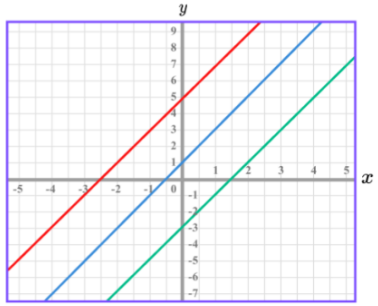
What will I learn about in this unit?

- Solving problems with graphs
- Finding lines parallel and perpendicular to others
- Solving simultaneous equations
- Plotting inequalities in two dimensions

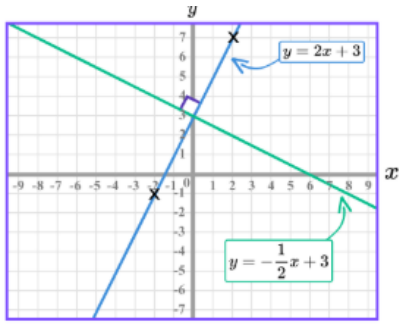
Where does this lead?

- Quadratic graphs (A12)
- Non-linear simultaneous equations, non-linear inequalities (A14)
- Tangents and normal (A15)
- Gradient of a curve (A16)
- Using graphs to represent complex problems (A Level Maths)

Parallel Lines have the **same gradient**.



Perpendicular lines meet at a right-angle, their lines have **gradients** that are the **negative reciprocal** of each other.



The line $y = 2x + 3$ has a gradient of 2

The line $y = -\frac{1}{2}x + 3$ has a gradient of $-\frac{1}{2}$

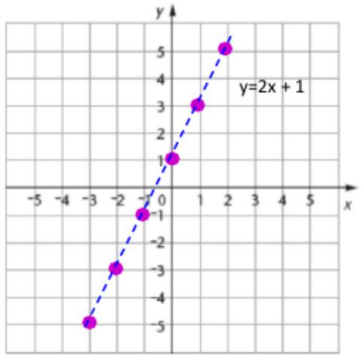
$2 \times -\frac{1}{2} = \frac{-2}{2} = -1$ The gradients multiply to give -1

How to complete a table and plot a straight line graph.

1) Complete the table by substituting the x values into the equation y = 2x + 1.
(This will give you the corresponding y values)

X	-3	-2	-1	0	1	2
y	-5	-3	-1	1	3	5

2) Plot the co-ordinates: (2, 5) (1, 3) (0, 3) (-1, -1) (-2, -3) (-3, -5)



Keyword/Skill	Definition/Tips
Linear	A graph that makes a straight line. Often written in the form y = mx + c
Graph	A drawing or diagram used to show information.
Distance	A measurement of how far.
Time	Minutes, Seconds, Hours, Days, Weeks, Months etc.
Coordinate	Shown as a pair (2, 4) to show a position on a set of axes.
Quadrant	Any of the 4 areas created by the x and y axes.
Real-Life Graphs	A graph that shows events of real life – distance and time.
Gradient	How steep a line is.
Y-Intercept	Where the line crosses y-axis
Function	A mathematical relationship between two values.
Solution	A value that makes an equation true.
Parallel	When two lines are always the same distance apart and never meet.

Year 11 – Maths – Unit A10 – Advanced Linear Graphs & Equations

In order to be able to solve simultaneous equations you will need to be able to **solve linear**
You also need to be able to use **substitution.**

Elimination Method

Example 1

Solve the equation: 6x + y =15 and 4x + y =11
It is useful to label the equations to help with method.

6x + y = 15 (1)
4x + y = 11 (2)

The y-terms are an **identical pair** (y in both equations has the same coefficient, positive 1). To get to zero with an identical pair we need to subtract them (1-1=0) this eliminates the y's.

6x + y = 15 (1)
4x + y = 11 (2)

Equation (1) minus equation (2)

2x = 4
x = 2 ÷ 2

Substitute x = 2 into one of the original equations.
So substitute x = 2 into equation (2)

8 + y = 11
y = 3 Solve

You can then test the solutions by substituting values found back into the original equations

Example 2

5x + y = 22 (1)
2x – y = 6 (2)

The y-terms are a **zero pair** (positive 1 y and negative 1 y). To get to zero with a zero pair we need to add them (-1+1=0) this eliminates the y's.

5x + y = 22 (1)
2x – y = 6 (2) Add

7x = 28
x = 4 ÷ 7

Substitute x = 4 into one of the original equations,
So substitute x=4 into equation (1)

20 + y = 22
y = 2 Solve

Sometimes you will have to change one or both of equations to get an identical or zero pair.

Example 3

Solve these equations:
4x + 3y = 27 (1)
5x – 2y = 5 (2)

Here, none of the coefficients are the same, so we need to scale both equations to get an identical or zero pair.

Equation (1) x 2
Equation (2) x 3

We can make a zero pair by scaling both equations to have 6 and -6 as the coefficients of y.

8x + 6y = 54 (3)
15x – 6y =10 (4)

Label new equations (3) and (4)

Eliminate by adding (3) + (4)

23x = 69
x = 3 ÷ 23

Substitute into equation (1)=
12 + 3y = 27
3y = 15
y = 5 Solve

Example 4- Apply to solve problems

Three chews and four biscuits cost 72p. Five chews and two biscuits cost 64p. What would three chews and five bubblies cost?

You need to set up two simultaneous equations for c and b and then solve them.

3c + 4b = 72
5c + 2b = 64

Solve the simultaneous equations as in examples 1, 2, and 3

c = 8
b = 12
Use these answers to calculate 3 chews = 3 x 8 = **24p**
5 bubblies= 5 x 12 = **60p**

Keyword/ Skill	Definition/tip
Simultaneous Equation	A pair of equations with two unknown variables. Both equations need to be solved at the same time (simultaneously)
Eliminate	To remove a variable in order to help solve the equation.
Substitution	When a letter in an equation, expression or formula is replaced by a number, we have substituted the number for the letter.
Variable	A symbol for a number that we don't know yet. Often this is a letter such as x or y.
Coefficient	The number in front of an unknown quantity (the letter) in an algebraic term.

Exam Tips

- You will gain 1 mark by correctly starting a process to eliminate a coefficient.
- Check your answers be substituting both values into an equation to see if it works.

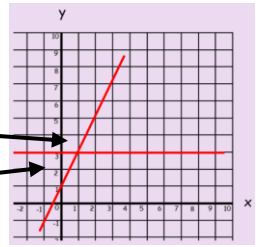
Year 11 – Maths – Unit A10 – Advanced Linear Graphs & Equations

Solving Simultaneous Equations Graphically

y = 2x + 1
y = y = 3

Step 1- Draw the line y = 2x + 1
Step 2- Draw the line y = 3
Step 3 - Your solution is the coordinates where the lines cross

Coordinates = (1, 3)
x = 1
y = 3



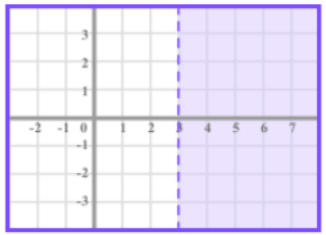
Plotting Inequalities in 2D

Inequalities on a graph allow us to visualise the regions that satisfy one or more inequalities.

To do this we use the linear equations to plot straight line graphs using either a solid line or a dashed line.
A **solid line** means the line is **included**.
A **dashed** or **dotted line** means the line is **not included.**
This is similar to using the solid (or closed) circle and open circles when displaying inequalities on a number line.

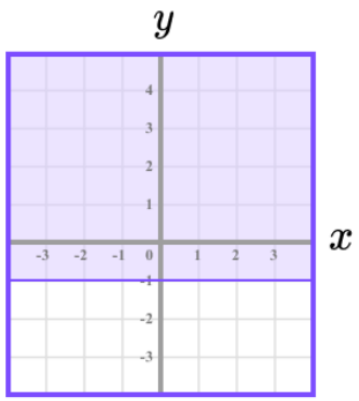
$x > 3$

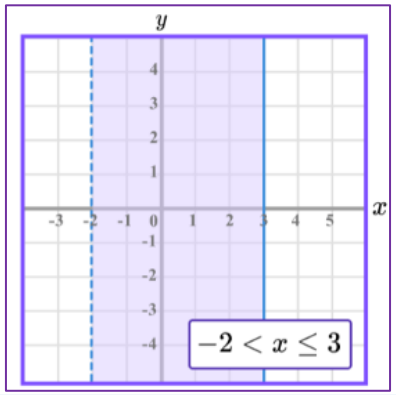
We use a dashed line for $x = 3$ and can shade the region required to the right of the line.



$y \geq -1$

We need y to be large to make it greater than -1. This means the region required is above the line.





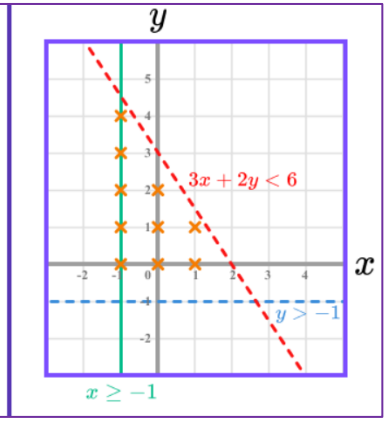
Systems of inequalities

For a system of inequalities, you need to draw the regions that satisfy all of the inequalities stated.
For example,

We can mark with a cross (x) all integer coordinates that satisfy the inequalities

$3x + 2y < 6$
$x \geq -1$
$y > -1$

We must remember that the points on dashed lines can not be included.



Keyword/ Skill	Definition/tip
Simultaneous Equation	A pair of equations with two unknown variables. Both equations need to be solved at the same time (simultaneously)
Eliminate	To remove a variable in order to help solve the equation.
Substitution	When a letter in an equation, expression or formula is replaced by a number, we have substituted the number for the letter.
Variable	A symbol for a number that we don't know yet. Often this is a letter such as x or y.
Coefficient	The number in front of an unknown quantity (the letter) in an algebraic term.

<u>Other topics/Units this could appear in</u>:

- Simultaneous equations
- Algebra and functions
- Statistical distributions

<u>Exam Tips</u>

- If you need to draw your own straight line graph you will gain marks for that.
- If the graph has been drawn for you LOOK at where the lines cross.

Year 11 – Maths – Unit GM4 – Congruence & Similarity

My mathematical journey

What do I need to remember from before?

Multiplying to scale (NP3, NP10)

Reflective and rotational symmetry (GM2)

Properties of shapes (GM2)

What will I learn about in this unit?

Congruence and similarity

Congruent transformations: translation, reflection, rotation

Similar transformations: enlargement

Where does this lead?

Trigonometry (GM5)

Area and volume in similar solids (GM8)

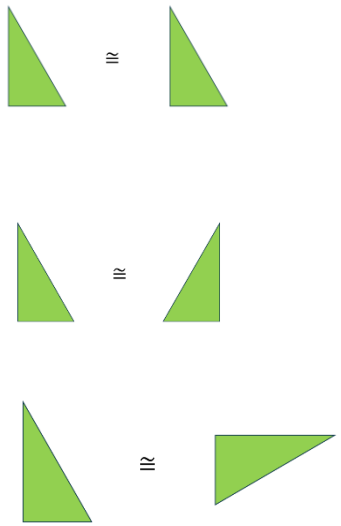
Problems with congruence and similarity (GM11)

Transforming graphs (A15)

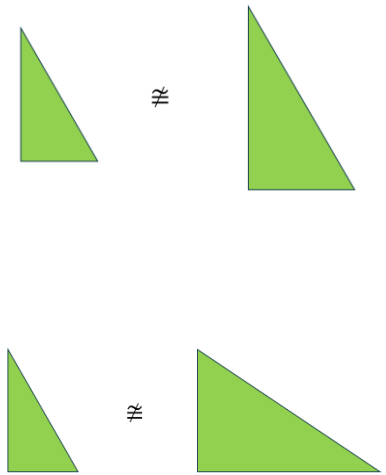
Congruency

We use the word congruent to describe shapes that are identical. Congruent shapes are the same size and shape, but they might face in a different direction or have a different orientation.

CONGRUENT

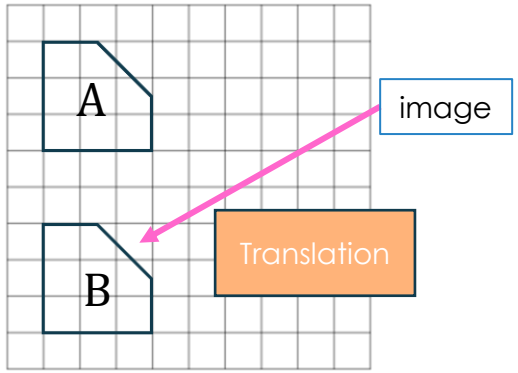


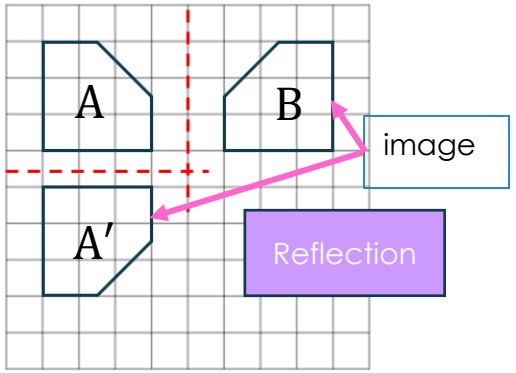
NOT CONGRUENT

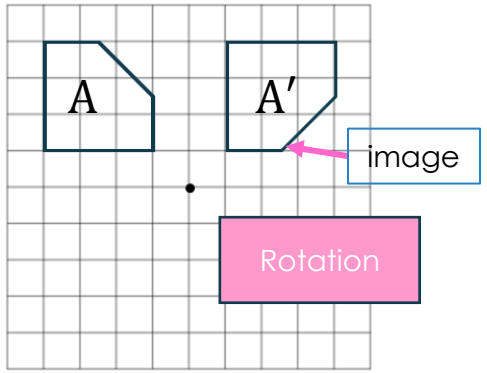


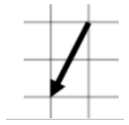
Transformations

When we transform a shape, we change something about it.
There are three congruent transformations we can do to shapes.
We can label shapes with capital letters, e.g. A.
We can label their images with different letters, e.g. B,
or with inverted commas, e.g. A′.







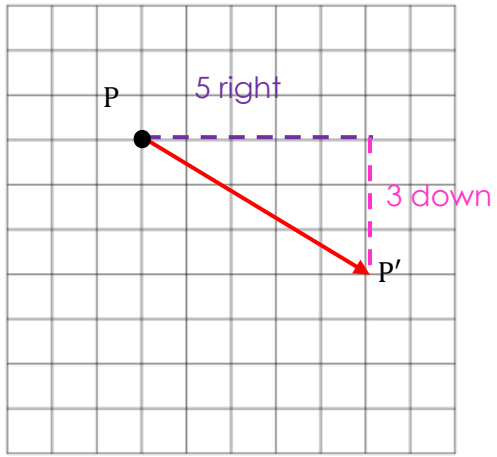
Keyword/Skill	**Definition/Tips**
Congruent	Identical in size and shape, but not necessarily orientation or direction. ≅ is the symbol used for congruency.
Transformation	A mathematical change, using translation, rotation, reflection or enlargement.
Image	A shape after a transformation has happened.
Vector	A mathematical object that tells you how far to move and in what direction it can be shown with an arrow or with column notation. $\begin{pmatrix}-1\\-2\end{pmatrix}$ 
Similar	Same shape, all angles the same, but one an enlargement of the other (all corresponding sides in the same ratio).

Year 11 – Maths – Unit GM4 – Congruence & Similarity

Translation

We can describe a transformation by giving clear instructions for it.

Translation involves shifting a shape left/right and up/down, with no turning or flipping.

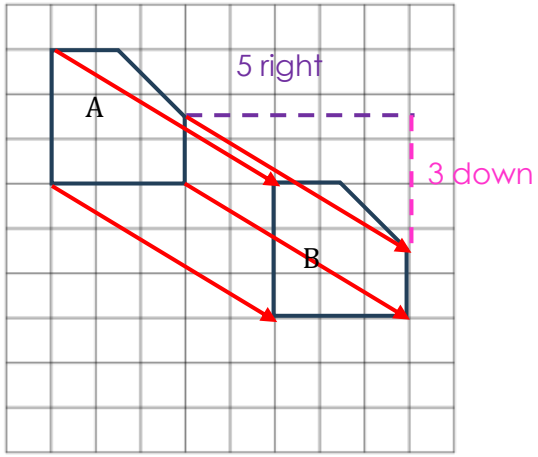


We use column vectors to describe translations of points.

Point P has been translated

5 right and 3 down

The translation vector is $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$.



We use column vectors to describe translations of shapes.

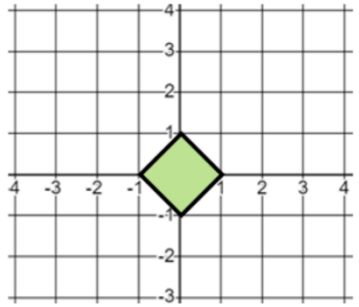
Every part of shape A has been translated

5 right and 3 down

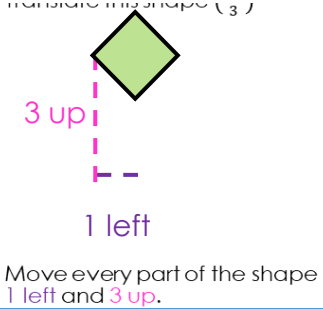
The translation vector is $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$

This works the same on graphs.

Translate this shape $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

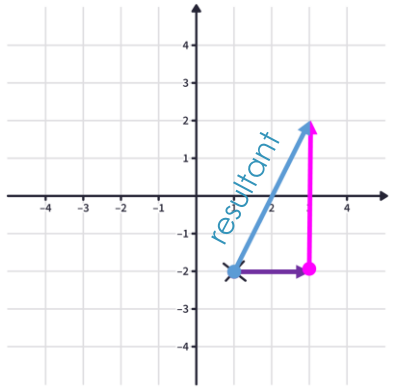


Move every part of the shape 1 left and 3 up.



Move every part of the shape 1 left and 3 up.

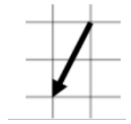
Column Vectors



We can also show vectors in two-dimensions.
2 units to the right, followed by 4 units up is the same as this diagonal movement.
This the resultant vector in a column vector.

Movement horizontally: 2 right → $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ ← Movement vertically: 4 up

The top number is horizontal movement (right is positive and left is negative).
The bottom number is vertical movement (up is positive and down is negative.)

Keyword/Skill	**Definition/Tips**
Congruent	Identical in size and shape, but not necessarily orientation or direction. ≅ is the symbol used for congruency.
Transformation	A mathematical change, using translation, rotation, reflection or enlargement.
Image	A shape after a transformation has happened.
Vector	A mathematical object that tells you how far to move and in what direction it can be shown with an arrow or with column notation. $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$ 
Similar	Same shape, all angles the same, but one an enlargement of the other (all corresponding sides in the same ratio).

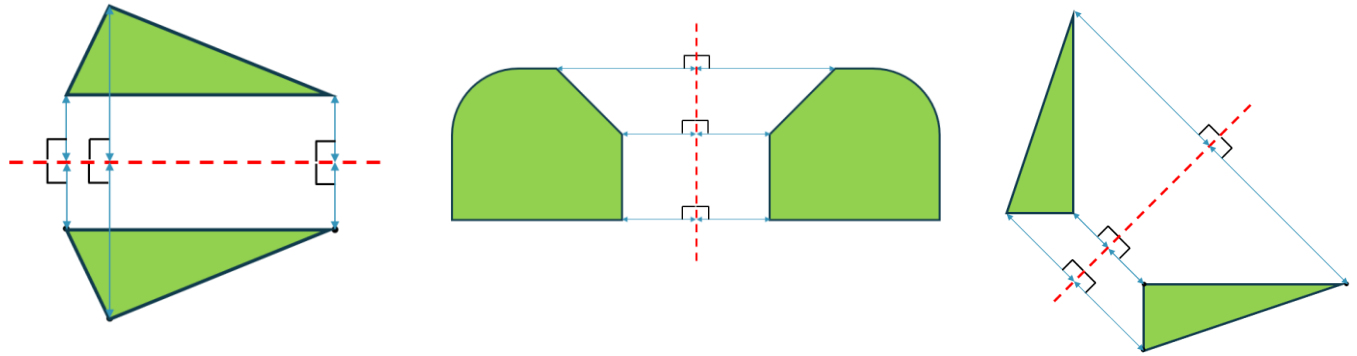
Year 11 – Maths – Unit GM4 – Congruence & Similarity

Reflection

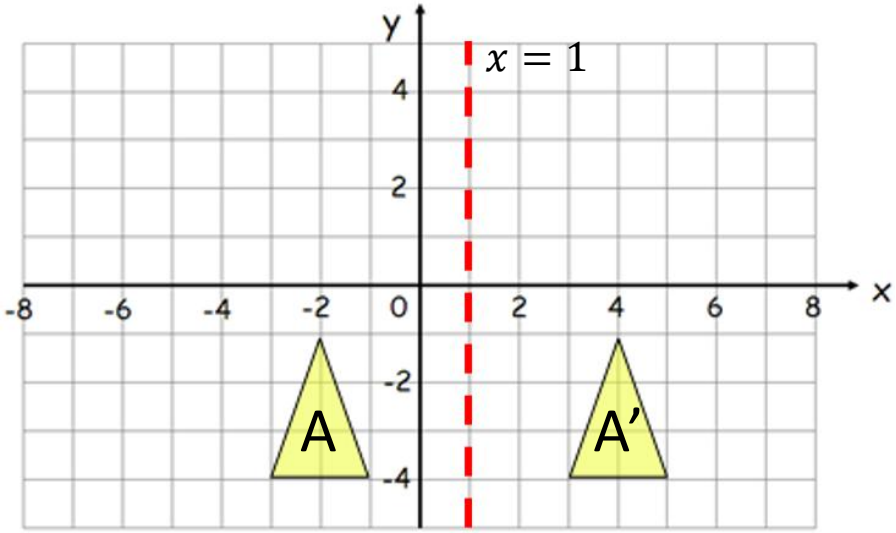
We can describe a transformation by giving clear instructions for it.

Reflection can be pictured as 'flipping' a shape over a mirror line.

The shapes are the same distance away from the mirror line.



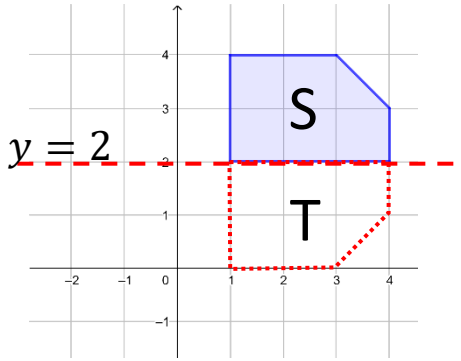
Reflect shape A in the mirror line.



This triangle has been reflected in the dotted line of reflection.

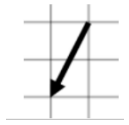
The line of reflection has the equation x = 1 because it passes through 1 on the x-axis and all its x-coordinates will be 1.

Reflect shape S in the line $y = 2$.



This shape has been reflected in the dotted line of reflection.

The line of reflection has the equation y = 2 because it passes through 2 on the y-axis and all its y-coordinates will be 2.

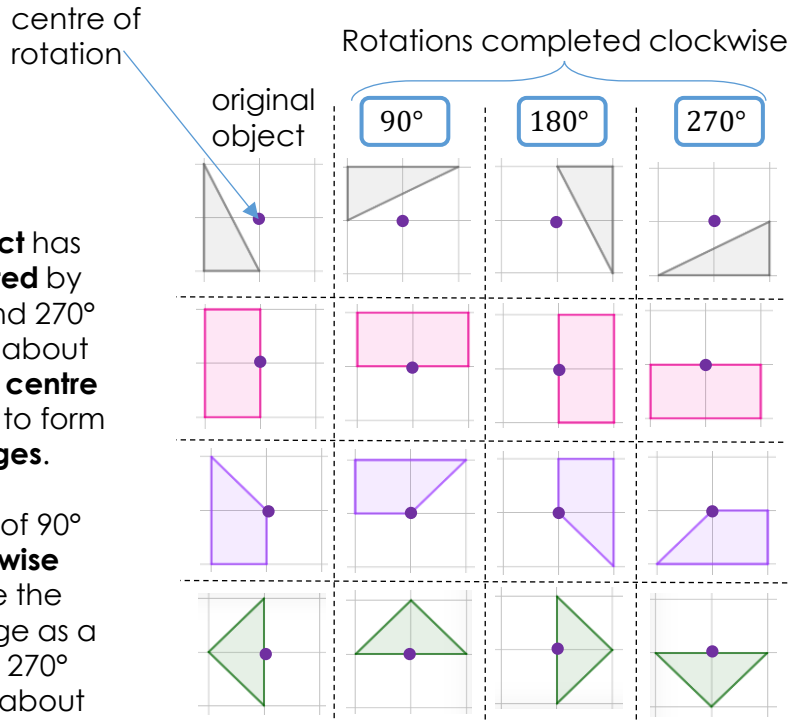
Keyword/Skill	**Definition/Tips**
Congruent	Identical in size and shape, but not necessarily orientation or direction. ≅ is the symbol used for congruency.
Transformation	A mathematical change, using translation, rotation, reflection or enlargement.
Image	A shape after a transformation has happened.
Vector	A mathematical object that tells you how far to move and in what direction it can be shown with an arrow or with column notation. $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$ 
Similar	Same shape, all angles the same, but one an enlargement of the other (all corresponding sides in the same ratio).

Year 11 – Maths – Unit GM4 – Congruence & Similarity

Rotation

We can describe a transformation by giving clear instructions for it.

Rotation can be pictured as moving in a circle around a fixed centre point.



Each **object** has been **rotated** by 90°, 180 and 270° **clockwise** about the purple **centre of rotation** to form these **images**.

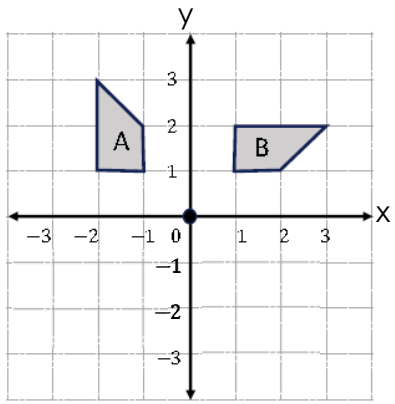
A rotation of 90° **anti-clockwise** would give the same image as a rotation of 270° clockwise about the same centre.

Describing Rotations

When we **describe** this type of transformation, we state that from A to B is:

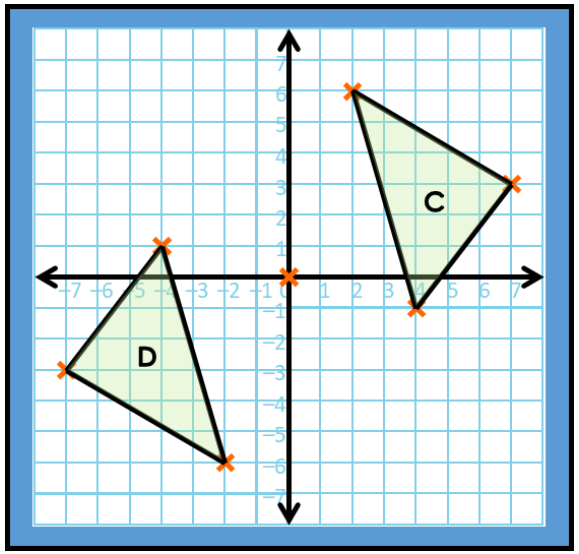
- A rotation
- From centre (0,0)
- By 90° clockwise

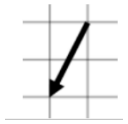
We must always give all three pieces of information to fully describe a rotation.



Triangle C has been rotated about the origin (0,0) by 180° to give triangle D.

The direction does not matter because 180° is a half turn, and a half turn clockwise has the same effect as a half turn anti-clockwise.



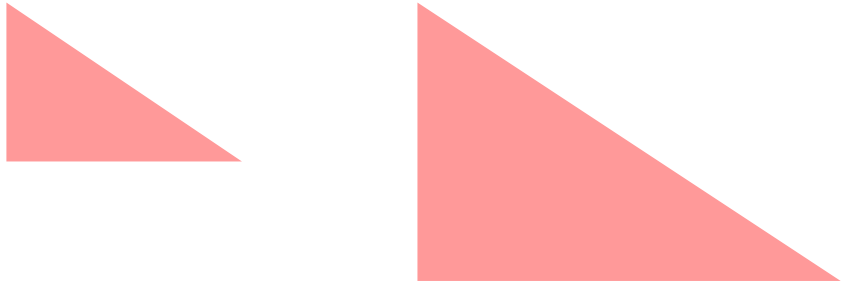
Keyword/Skill	**Definition/Tips**
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Image	A shape after a transformation has happened.
Vector	A mathematical object that tells you how far to move and in what direction it can be shown with an arrow or with column notation. $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$ 
Similar	Same shape, all angles the same, but one an enlargement of the other (all corresponding sides in the same ratio).

Similarity

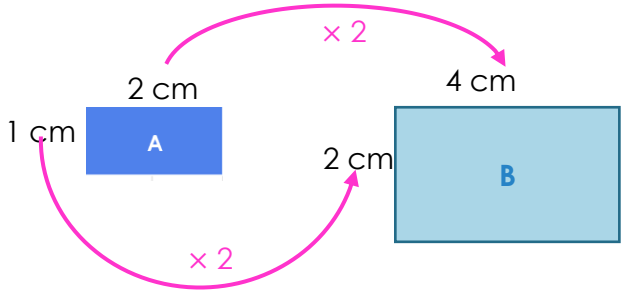
Here is a shape.

We can enlarge the shape by multiplying all its side lengths by the same scale factor.

The original shape and the enlarged shape are called similar shapes.



For two shapes to be similar, they must have
- ✓ all angles the same
- ✓ all sides in the same ratio

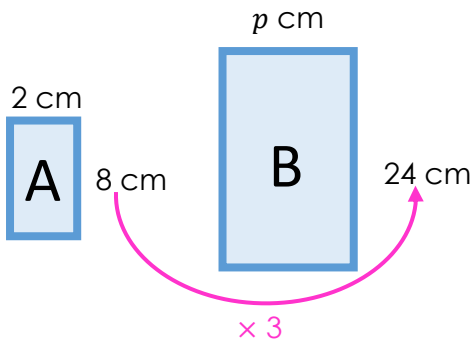


- ✓ all angles the same
- ✓ all sides in the same ratio

} Shapes A and B are similar.

Rectangle A & B are similar.

To find the scale factor, look at the corresponding sides.

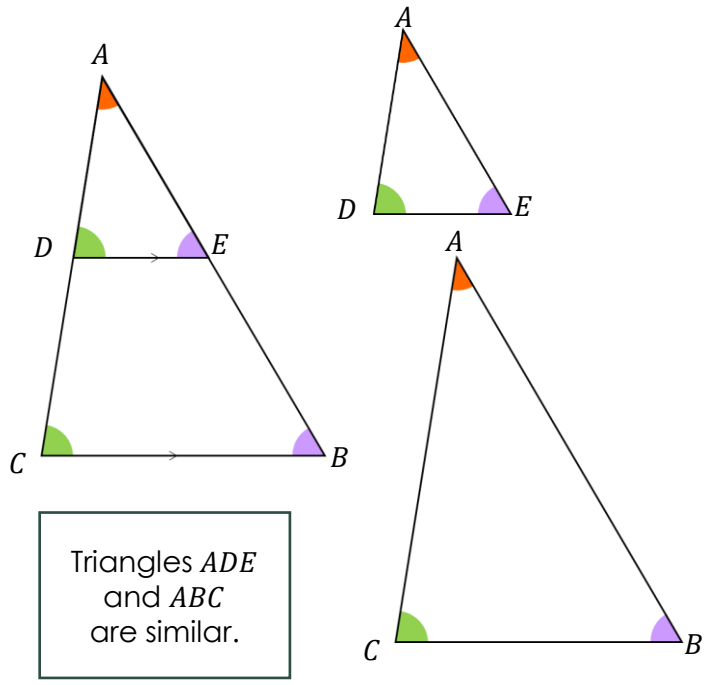


That means to find the missing side we need to multiply by the scale factor.

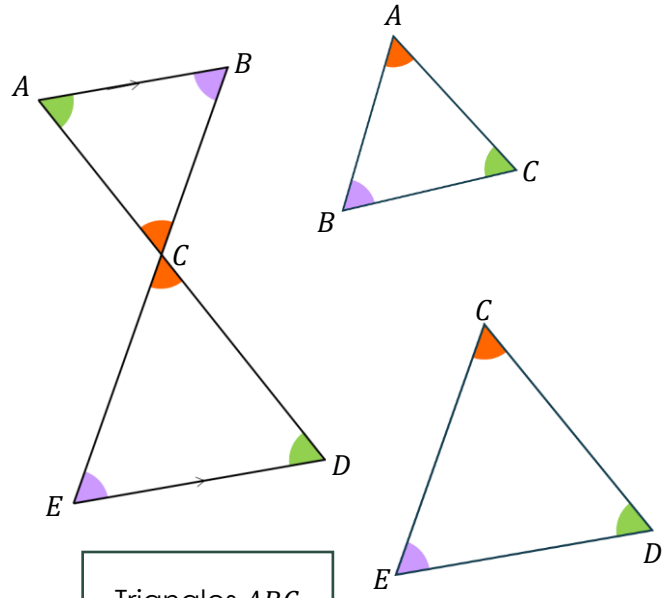
$$p = 2cm \times 3 = 6cm$$

Sometimes, similar triangles can be hidden.

We can use our knowledge of angles in parallel lines to find them.



Triangles ADE and ABC are similar.



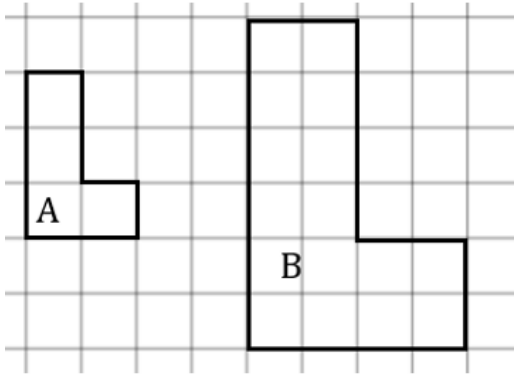
Triangles ABC and CDE are similar.

Enlargement

Enlargement is a type of transformation.

It is not a congruent transformation like translation, reflection and rotation.

It is a similar transformation, since object and image are similar.



The two shapes are enlargements of each other.

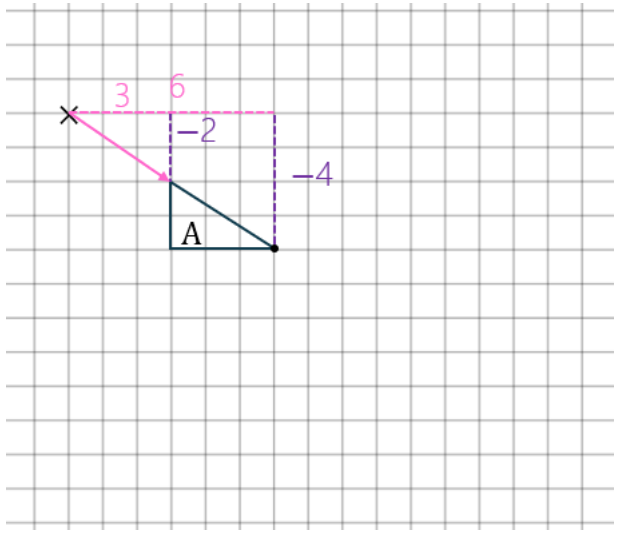
B is an enlargement of A (scale factor: 2)

A is an enlargement of B (scale factor: $\frac{1}{2}$)

A scale factor controls the size of an enlarged image.

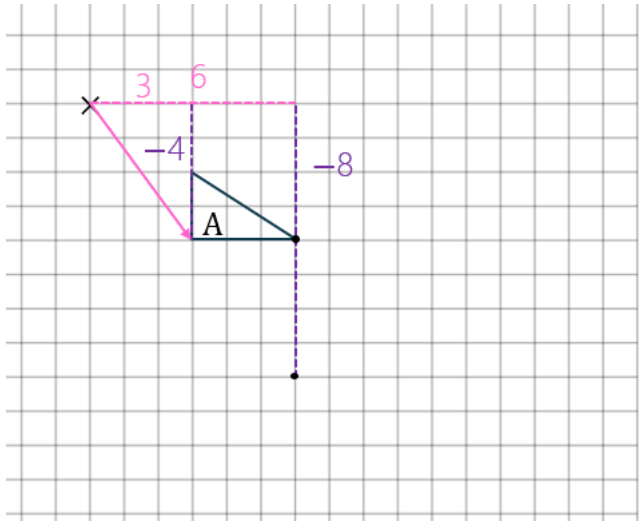
A centre of enlargement controls the position of an enlarged image.

Let's enlarge triangle A by scale factor 2, from the centre marked ×.



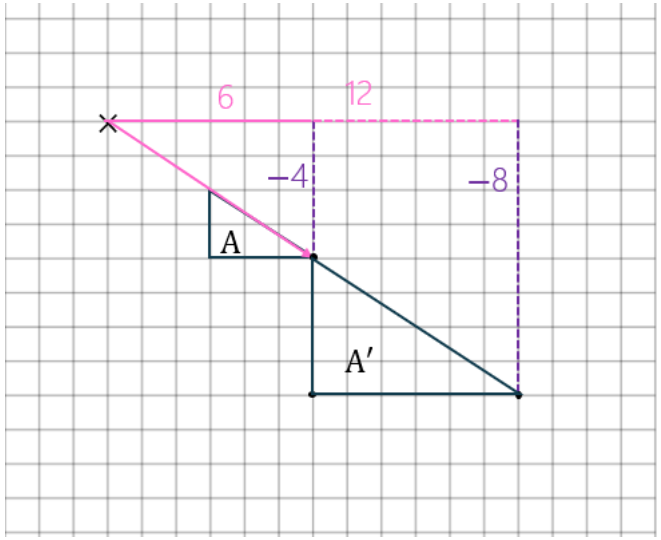
Scale factor is 2, so we multiply all distances by 2.

$\begin{pmatrix} 3 \\ -2 \end{pmatrix} \times 2 = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$



Scale factor is 2, so we multiply all distances by 2.

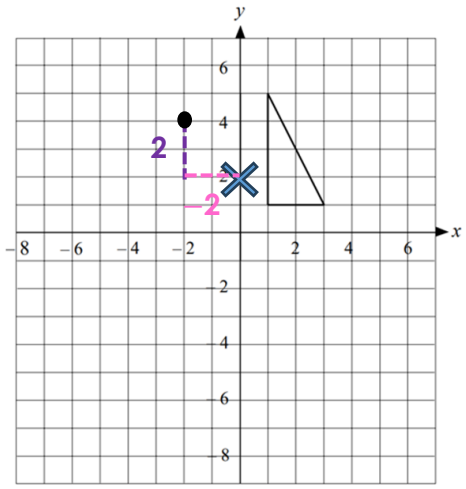
$\begin{pmatrix} 3 \\ -2 \end{pmatrix} \times 2 = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$



Scale factor is 2, so we multiply all distances by 2.

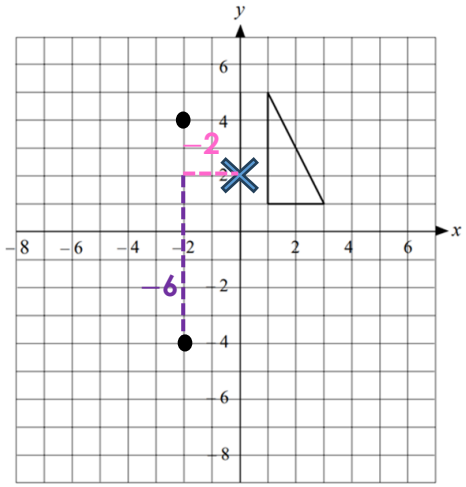
$\begin{pmatrix} 3 \\ -2 \end{pmatrix} \times 2 = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

A negative scale factor makes you enlarge in the opposite direction.

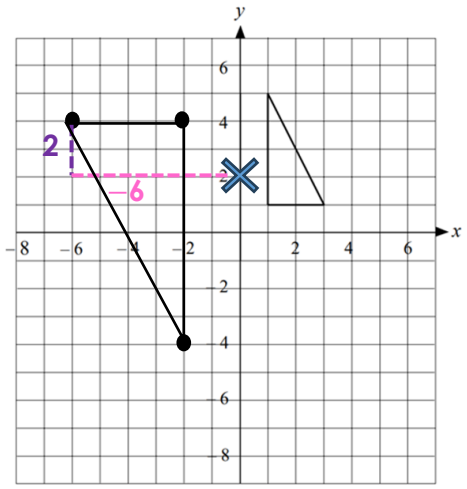


$\begin{pmatrix}1\\-1\end{pmatrix} \times -2 = \begin{pmatrix}-2\\2\end{pmatrix}$

Let's enlarge triangle A by scale factor −2, from the centre marked ×.

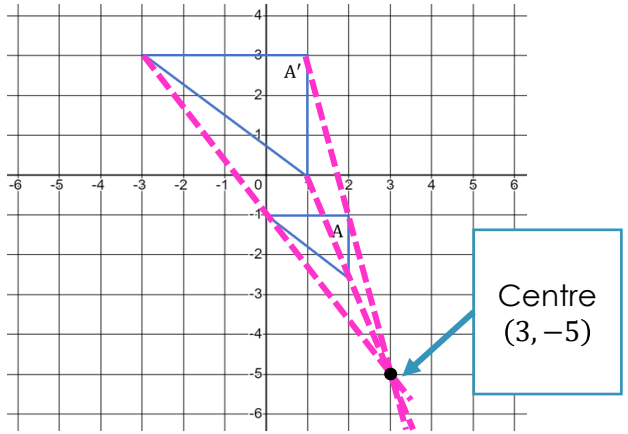


$\begin{pmatrix}1\\3\end{pmatrix} \times -2 = \begin{pmatrix}-2\\-6\end{pmatrix}$

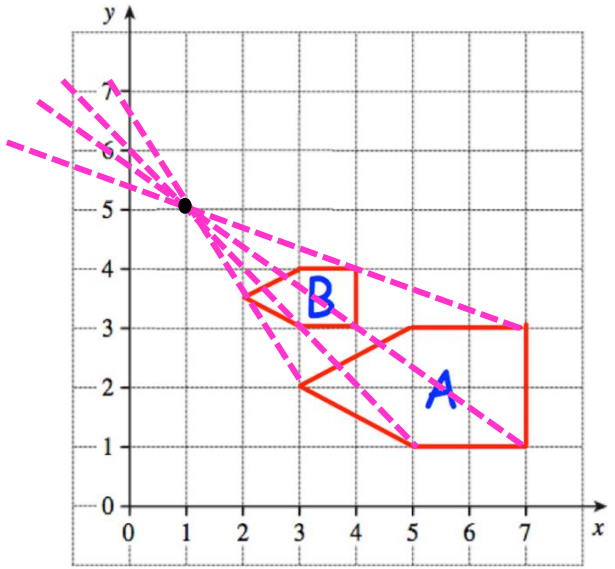


$\begin{pmatrix}3\\-1\end{pmatrix} \times -2 = \begin{pmatrix}-6\\2\end{pmatrix}$

To find the centre of an enlargement, draw rays that join corresponding vertices on a shape and its image. The rays meet at the centre.



A has been enlarged with a scale factor of 2 from point $(3, -5)$.



Make sure you read the question carefully to avoid choosing the wrong transformation.

The transformation of A to B is:

Enlargement, centre $(1, 3)$, Scale Factor: $\frac{1}{2}$

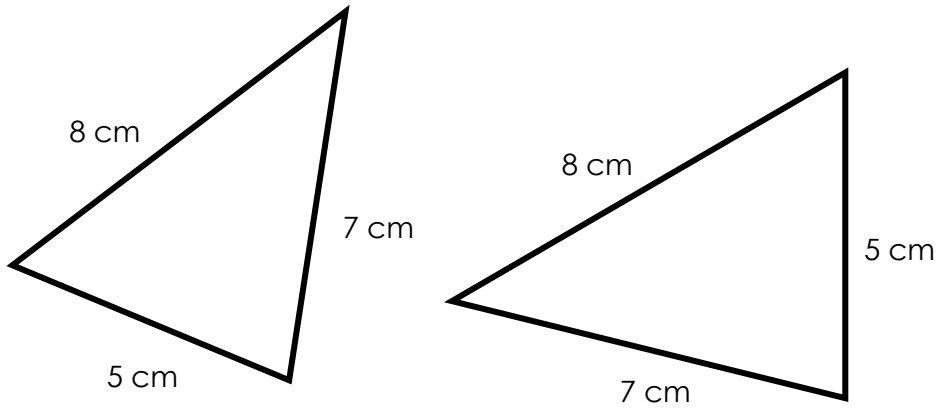
The transformation of B to A is:

Enlargement, centre $(1, 3)$, Scale Factor: 2

Conditions for Congruency

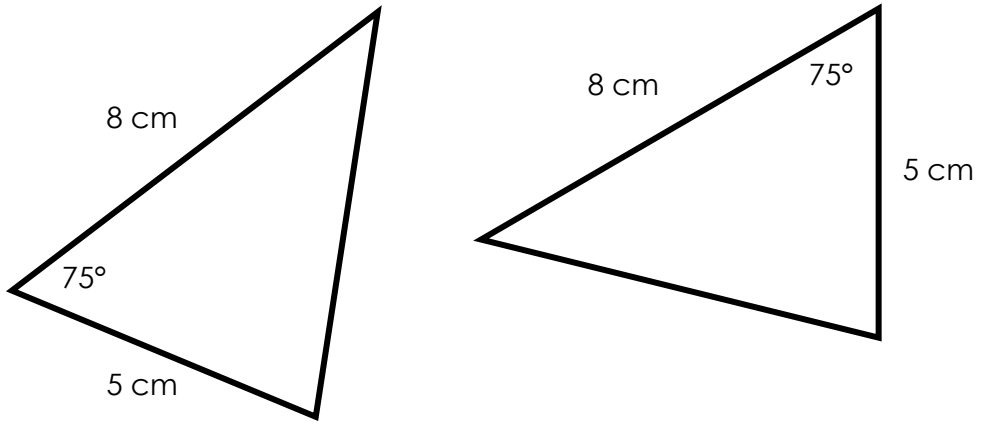
All sides the same: congruent

SSS



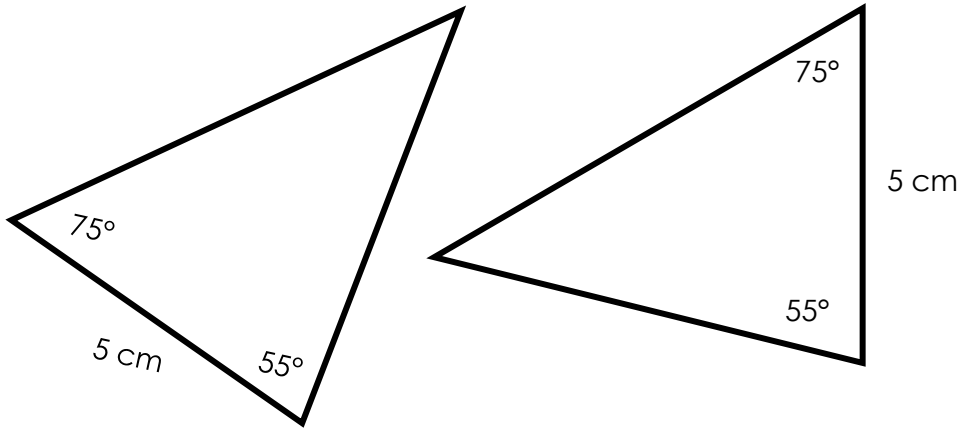
Two sides and the angle between: congruent

SAS



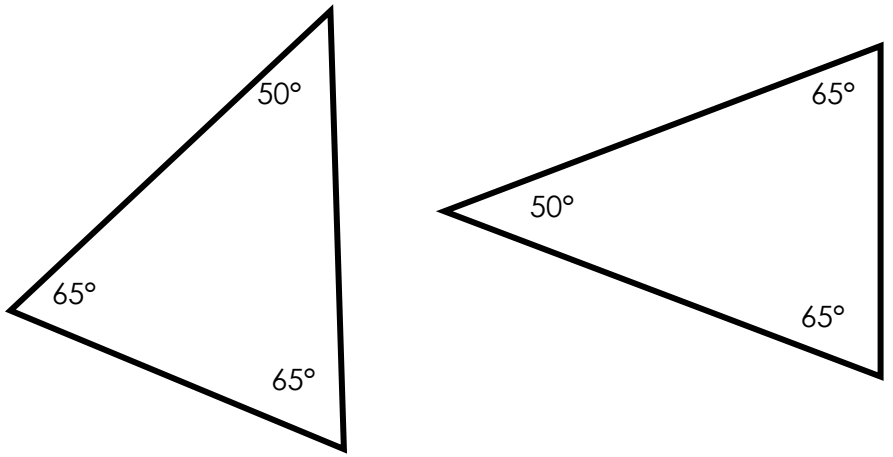
Two angles and the side between: congruent

ASA



All angles the same: similar

AAA



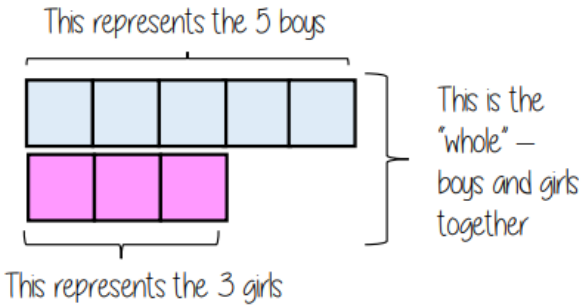
Year 11 – Maths – Unit NP11 – Ratio

Ratio Notation

A ratio is a relationship between two or more quantities. They are written in the form $a:b$, which is said "a to b".

For example, in a class, the ratio of boys to girls is $5:3$.

The bar model to the right is one way we can visualise this information.



My mathematical journey

What do I need to remember from before?	What will I learn about in this unit?	Where does this lead?
Multiplication and division; multiples and factors (NP3)	Using ratio notation	Combining ratios (NP13)
Writing values as a fraction; equivalent fractions (NP7)	Equivalent ratios and simplifying	Similar area and volume (GM8)
Ratio tables (NP10)	Ratios and fractions	Geometric sequences (A13)
	Finding values from parts or the whole	Advanced ratio (NP16)
		Vectors (GM10)

Simplifying Ratios and Equivalent Ratios

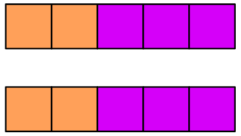
We can scale ratios up and down to make equivalent ratios or to simplify them.

O 

P 

Here we have some orange and purple tiles. They are in the ratio of $4:6$.

We can regroup these tiles into two equal groups of 2 orange tiles and 3 purple tiles.



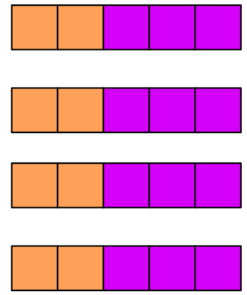
So, for every 2 orange tiles there are 2 purple tiles.

This can be written as the ratio $2:3$

$$4:6 \xrightarrow{\div 2} 2:3$$

We can produce the same result without the bar model, by identifying the HFC of the two parts of the ratio.

We could also make an equivalent ratio by repeating the pattern of orange and purple tiles.



We now have 8 orange tiles there are 12 purple tiles.

This can be written as the ratio $8:12$

$$4:6 \xrightarrow{\times 2} 8:12$$

We can produce the same result without the bar model, by multiplying both parts of the ratio by the same value

Year 11 – Maths – Unit NP11 – Ratio

Unit Ratios

You may be asked to express a ratio in the form $1:n$ or $n:1$.
These means you need to find an equivalent ratio where one of the parts is 1.

Example 1

Write the ratio $12:4$ in the form $n:1$

$12:4$ → $\div 4$ → $3:1$

In order to get the second number in the ratio down to 1, you would need to divide it by 4.
Therefore, we need to do the same thing to the first part of the ratio.

Example 2

Write the ratio $4:6$ in the form $1:n$

$4:6$ → $\div 4$ → $1:1.5$

This time we need the first number in the ratio to become 1, in order to do this you would need to divide it by 4.
Therefore, we need to do the same thing to the second part of the ratio.

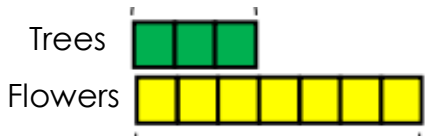
In this example, our result is no longer an integer, this is ok when we are asked to write it in the for $1:n$ or $n:1$.

Ratios as Fractions

You may also be asked to express a ratio as a fraction.

Example

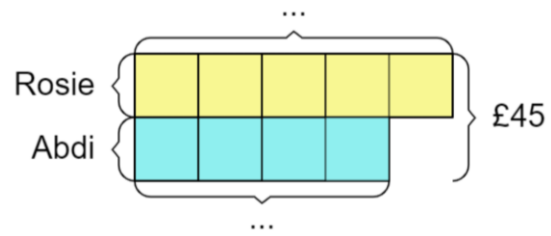
The ratio of trees to flowers is $3:7$. Express this as a fraction.



As a fraction

$$\frac{number\ of\ parts\ in\ group}{total\ number\ of\ parts} = \frac{3}{10}$$

Finding the Value of Parts of a Ratio



Finding parts of a ratio, give the whole

Rosie and Abdi share £45 in the ratio 5 : 4.
We could use a bar model to find how much each of them receives.

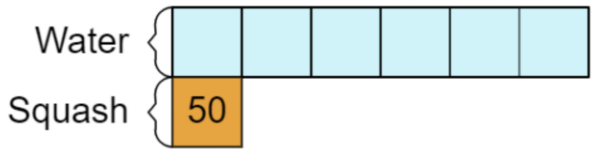
There are 9 parts in the ratio in total, so £45 needs to e shared equally into the bar model.

$$45 \div 9 = 5$$

Therefore, each box is worth £5.
So, Rosie get $5 \times 5 = £25$ and
Abdi gets $4 \times 5 = £20$

Finding parts of a ratio, given another part

Jamal is making orange squash.
He needs to mix squash : water in the ratio 1 : 6.
He has already poured 50ml of squash into his glass.
How much water should he use?
In this diagram we can see one box is worth 50ml, this means every box must be worth 50ml.
So, we need $6 \times 5 = 30$ml of water.



Finding parts of a ratio, given the difference

Zainab and Omar are sharing some sweets in the ratio of their ages, 7 : 9.
Omar receives 4 more sweets than Zainab.
We could use a bar model to help us find how many sweets each of them receives.



In this example, you can see that the two extra boxes in Omar's row are worth 4 in total.
Therefore, each box must be worth 2.
Which means, Zainab has $7 \times 2 = 14$ sweets
And Omar has $9 \times 2 = 18$ sweets.

Year 11 – Maths – Unit NP12 – Standard Form

Index form makes it easier to write large and small powers of 10.

$10\,000 = 10^4$
$100\,000 = 10^5$
$1\,000\,000 = 10^6$
$0.1 = 10^{-1}$
$0.01 = 10^{-2}$
$0.001 = 10^{-3}$

$$3.04 \times 10^8 = 304000000$$

Standard form — Ordinary Number

Standard index form, known as standard form, makes it much easier to write large and small numbers. A number is in standard form when it is written as $A \times 10^n$, where A is a number between 1 and 10 and n can be any integer. The power of 10, n, tells us how many place value columns to move up or down.

My mathematical journey

What do I need to remember from before?
Place value, multiplication and division by powers of 10 (NP1)
Operations (NP2, NP3, NP4)
Laws of indices (NP5)

What will I learn about in this unit?
Writing large and small numbers in standard form.
Calculating with numbers in standard form.

Where does this lead?
Compound units (NP13)
Index laws (NP15)
Exponential growth (NP16)
You will apply this knowledge in Physics, Chemistry and Biology

Keyword/Skill	Definition/Tips
Decimal Numbers/ Ordinary Numbers	The numbers we use in everyday life are decimal numbers, because they are based on 10 digits (0,1,2,3,4,5,6,7,8 and 9). 29000 is an example of an ordinary number.
Standard Form	A way of writing very large numbers or very small numbers using a number between 1 and 10, multiplied by a power of 10.
Power	The number of times a base number is multiplied by itself.
Index	A small number placed on the upper-right of a base number to inform how many times to multiply by itself.

Converting Numbers into Standard Form

Convert 3920000 into standard form.

We need a decimal point between the first and second digit.

$$3920000.$$

We need to move the digits 6 place value columns.
Therefore $3920000 = 3.92 \times 10^6$

Convert 0.00081 into standard form.

We need a decimal point between the first non-zero digit and the second digit after that.

$$0.00081$$

We need to move the digits 4 place value columns.
Therefore $0.00081 = 8.1 \times 10^{-4}$

Converting Standard Form into Ordinary Numbers

Convert 3×10^3 into an ordinary number.
We can see how many times we need to multiply by 10.
$3 \times 10^3 = 3 \times 10 \times 10 \times 10 \times 10$
$3 \times 1000 = 3000$
$3 \times 10^3 = 3000$

We can see this using a place value grid too.

1,000	100	10	1	0.1	0.01	0.001	0.0001
Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten Thousandths
			3				
		3	0				
	3	0	0				
3	0	0	0				

Convert 2.4×10^{-3} into an ordinary number.
Multiplying by 10^{-1} is the same as dividing by 10
(You will have saw this in NP1).
$2.4 \times 10^{-3} = 2.4 \times 10^{-1} \times 10^{-1} \times 10^{-1}$
$2.4 \times 10^{-3} = 2.4 \div 10 \div 10 \div 10$
$2.4 \times 10^{-3} = 0.0024$

We can see this using a place value grid too.

1,000	100	10	1	0.1	0.01	0.001	0.0001
Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten Thousandths
			2	4			
			0	2	4		
			0	0	2	4	
			0	0	0	2	4

Year 11 – Maths – Unit NP12 – Standard Form

Adjusting into Standard Form

84×10^5 is a number not in standard form (84 is not between 1 and 10).

To convert into standard form, we can adjust each side of the number

We can first divide 84 by 10

$$84 \times 10^5$$

$$\div 10$$

$$8.4 \times 10^5$$

The number is now 10 times smaller, so we need to adjust the power of 10 to keep it balanced.

$$8.4 \times 10^5$$

$$\times 10$$

$$8.4 \times 10^6$$

Now the number is in standard form. $84 \times 10^5 = 8.4 \times 10^6$

Another example:

0.0801×10^9

We need to multiply 0.0801 by 100. $100 = 10^2$

$$\times 10^2$$

8.01×10^9

$$\div 10^2$$

Now we need to divide by 10^2 to keep it balanced.

8.01×10^7

Multiplying Standard Form

We can multiply numbers in standard form without converting them into ordinary form.
We use commutativity to reorder the multipliers.

Example:

$$(2 \times 10^4) \times (4 \times 10^5)$$

Remember to add the powers when multiplying two powers!

$$= 2 \times 4 \times 10^4 \times 10^5$$

$$= 8 \times 10^9$$

Make sure to check if your answer is in standard form (if the question asks for it).

Example:

$$3.1 \times 10^7 \times 5 \times 10^{-2}$$

$$= 3.1 \times 5 \times 10^7 \times 10^{-2}$$

$$= 15.5 \times 10^5$$

$$\div 10$$

$$= 1.55 \times 10^5$$

$$\times 10$$

$$1.55 \times 10^6$$

Dividing Standard Form

We can divide standard form in the same sort of method as multiplying.

$$8.8 \times 10^{14} \div 4 \times 10^6$$

We can write the division as a fraction.

$$= \frac{8.8 \times 10^{14}}{4 \times 10^6}$$

We can write this as a multiplication of two fractions.

$$= \frac{8.8}{4} \times \frac{10^{14}}{10^6}$$

Carry out the divisions and check it is standard form.

$$= 2.2 \times 10^8$$

Always make sure you convert back to standard form; most exam style questions will ask your answer to be in standard form with these questions.

$$3 \times 10^{11} \div 6 \times 10^3$$

$$= \frac{3 \times 10^{11}}{6 \times 10^3}$$

$$= \frac{3}{6} \times \frac{10^{11}}{10^3}$$

$$= 0.5 \times 10^8$$

$$\times 10 \qquad \div 10$$

$$= 5 \times 10^7$$

Year 11 – Maths – Unit NP12 – Standard Form

Adding & Subtracting Standard form

When we are adding or subtracting, we have seen they need to be in the same counting unit. There are two ways we can look at adding and subtracting standard form.

Method 1

You can convert standard form into ordinary numbers and then do the calculation.

Example:

$(4.8 \times 10^5) + (1.9 \times 10^4)$

Make sure you align the place value columns.

$$\begin{array}{r} 480000 \\ +\ 19000 \\ \hline 499000 \end{array}$$

We can convert our answer back intro standard form.

$$499000 = 4.99 \times 10^5$$

Example:

$1.09 \times 10^4 - 2.3 \times 10^2$

Make sure you align the place value columns.

$$\begin{array}{r} 10900 \\ -\ 230 \\ \hline 10670 \end{array}$$

We can convert our answer back into standard form.

$$10670 = 1.067 \times 10^4$$

Method 2

You can also add and subtract numbers in standard form by making the counting unit – the power of 10 – the same. The most efficient way to do this is to convert both into the larger power of 10.

Example:

$$(5 \times 10^5) + (2 \times 10^6)$$

$\div 10$ $\times 10$

$$= (0.5 \times 10^6) + (2 \times 10^6)$$

$$= (0.5 + 2) \times 10^6$$

$$= (2.5 \times 10^6)$$

Here our answer is in standard form, so we do not need to adjust it.

Example:

$$1 \times 10^{10} - 9 \times 10^9$$

$\div 10$ $\times 10$

$$1 \times 10^{10} - 0.9 \times 10^{10}$$

$$(1 - 0.9) \times 10^{10}$$

$$0.1 \times 10^{10}$$

$\times 10$ $\div 10$

$$1 \times 10^9$$

Problems with Standard Form

With mathematic questions, we need to be able to deduce which operation we are doing. Key words in the question tell us this information.

Example:

Spain has an area of $5.1 \times 10^5\ \text{km}^2$.
The area of Portugal is $9.2 \times 10^4\ km^2$.
What is the difference between the areas of the two countries?

The word that helps us here is **difference**. This means we need to subtract. You can use either method you have been shown here. I will use method 2.

$$5.1 \times 10^5 \ - \ 9.2 \times 10^4$$

$$= 5.1 \times 10^5 - 0.92 \times 10^5$$

$$= 4.18 \times 10^5$$

Look out for words which help us deduce the operation we need to do in worded questions.

SI Units

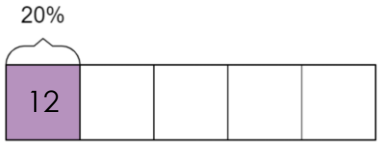
Unit name	Abbreviation	Measurement
metre	m	length
kilogram	kg	mass
second	s	time
ampere	A	electrical current
kelvin	K	temperature
mole	mol	number of particles
candela	cd	light intensity

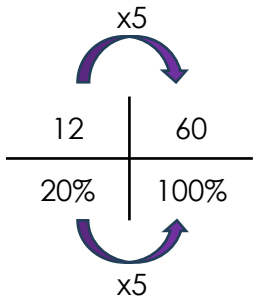
Year 11 – Maths – Unit NP13 – Advanced Proportion & Rates of Change

Reverse Percentages

If we know part of a number, we can work out the number.

If 20% of a number is 12, what is the number?





To calculate a percentage: $original \times multiplier = new$

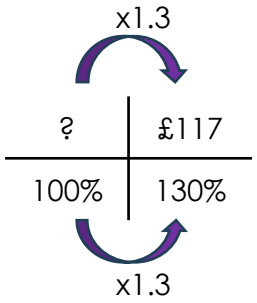
In questions like this, you need to decide if you have the original amount or the new amount.

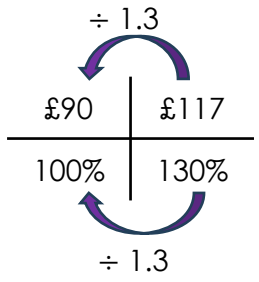
Bobby bought a pair of trainers.

He sold them for £117 and made a 30% profit.

How much did he originally pay for the trainers?

30% profit means a 30% increase. 100% + 30% = 130%





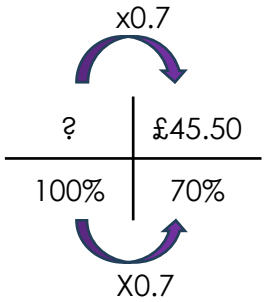
To return to the original whole: $original = \frac{new}{multiplier}$

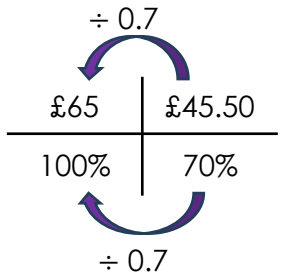
A jacket is reduced by 30% in a sale.

The sale price of the jacket is £45.50.

Work out the original price.

30% sale means a 30% decrease. 100% - 30% = 70%





My mathematical journey

What do I need to remember from before?

- Percentage multipliers (NP8, NP10)
- Proportional reasoning (NP10)
- Cartesian graphs (A6, A9, A10)
- Manipulating algebra (A5, A10)

What will I learn about in this unit?

- Reverse percentages
- Simple interest
- Direct and inverse proportion
- Compound units
- Combining ratios

Where does this lead?

- Compound interest (NP16)
- Proportion with squares, cubes and roots (NP16)
- Instantaneous and average rates of change (A16)
- Areas under graphs (A16)

Keyword/Skill	Definition/Tips
p.a.	Per annum, Latin for 'every year'.
Interest	An increase in money.
Depreciation	A decrease in money.
Direct proportion	As one amount increases, the other increases at the same rate.
k	The constant of proportionality, which is the rate of change.
$\propto$	Is proportional to.
Inverse proportion	As one amount increases, the other decreases at the same rate.
Density	A measure of how spread-out particles in an object are.
Pressure	A measure of how spread out a force is.
Speed	A measure of distance per unit of time.
Compound units	A measure combining two other measures.

Year 11 – Maths – Unit NP13 – Advanced Proportion & Rates of Change

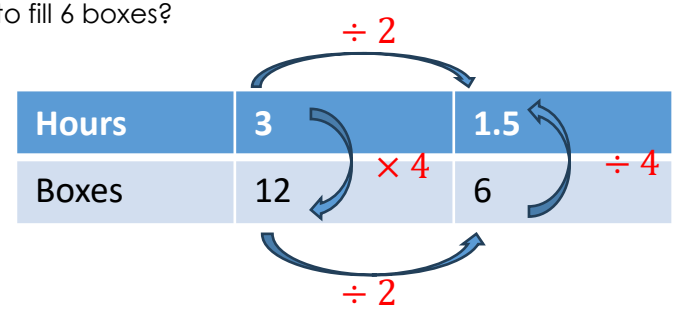
Direct Proportion

If y is directly proportional to x, we represent the relationship as follows $y \propto x$

Then, $y = kx$, where k is the constant of proportionality.

Solving Numerically Using Ratio Tables

It takes 3 hours to fill 12 boxes with food. How long does it take to fill 6 boxes?



You can use any of the relationships shown in the table above to find the answer, 1.5 hours.

Solving Algebraically

a is directly proportional to b.
a = 27 when b = 9 Find the value of a when b = 5

$a \propto b$
$a = kb$

Substitute the values for a and b from the question to find k
$27 = k \times 9$
$3 = k$
Rewrite and substitute k into the equation to solve
$a = 3b$
$a = 3 \times 5$
$a = 15$ when $b = 5$

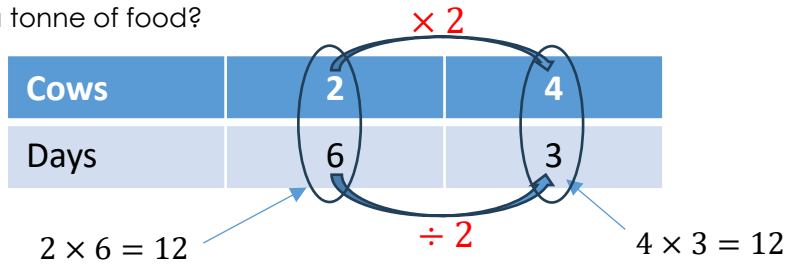
Inverse Proportion

If y is indirectly proportional to x, we represent the relationship as follows $y \propto \frac{1}{x}$

Then, $y = \frac{k}{x}$, where k is the constant of proportionality.

Solving Numerically Using Ratio Tables

2 cows eat a tonne of food in 6 days. How long would it take 4 cows to eat a tonne of food?



You can use any of the relationships shown in the table above to find the answer, 3 days.

Solving Algebraically

y is inversely proportional to x
y = 6 when x = 5. Find the value of y when x = 10

$y \propto \frac{1}{x}$
$y = \frac{k}{x}$

Substitute the values for x and y from the question to find k.
$6 = \frac{k}{5}$
$30 = k$
Rewrite and substitute k into the equation to solve
$y = \frac{30}{x}$

$y = \frac{30}{10}$
$y = 3$

Keyword/Skill	Definition/Tips
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k	The constant of proportionality, which is the rate of change.
$\propto$	Is proportional to.
Inverse proportion	As one amount increases, the other decreases at the same rate.
Density	A measure of how spread-out particles in an object are.
Pressure	A measure of how spread out a force is.
Speed	A measure of distance per unit of time.
Compound units	A measure combining two other measures.

Simple Interest

You can save money in a savings account with a bank or building society. The bank or building society gives you interest – a percentage of extra money on top of your original investment. For simple interest, work out the percentage.
Keep adding it on

A £200 loan earns 15% **simple** interest over 5 years. How much will be owed at the end of the 5 years?

Work out 15% of £200 = £30
£30 x 5 years = £150
£200 + £150 = £350

Year 11 – Maths – Unit NP13 – Advanced Proportion & Rates of Change

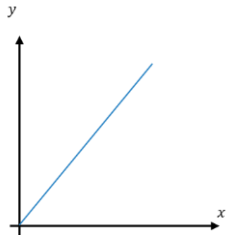
Direct Proportion

As one increases, the other increases at the same rate.

y is proportional to x

$y \propto x$

$y = kx$



$\times k$	x		
	y		

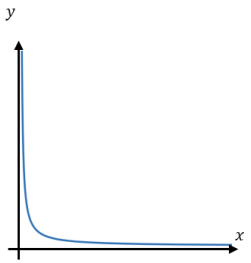
Inverse Proportion

As one increases, the other decreases at the same rate.

y is proportional to the reciprocal of x

$y \propto \frac{1}{x}$

$y = \frac{k}{x}$



x	$\times$	$\times$
y		
	k	k

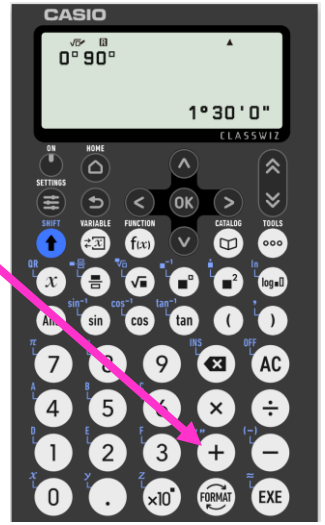
We can work with time on our calculator using SHIFT and this button.
We enter time in the form HOURS° MINUTES° SECONDS°

We can type in 90 minutes and use FORMAT to convert it to

1 hour and 30 minutes

or 1.5 hours

or $1\frac{1}{2}$ hours



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Pressure	A measure of how spread out a force is.
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Compound units	A measure combining two other measures.

$$Density = \frac{mass}{volume}$$

$$Pressure = \frac{force}{area}$$

$$Speed = \frac{distance}{time}$$

Year 11 – Maths – Unit NP13 – Advanced Proportion & Rates of Change

Density

Density is a measure of the amount of mass an object has per unit volume.

Density is a compound measure – made up of two other measures.

Its units are a combination of mass and volume.

$$\text{Density} = \frac{\text{mass (kg)}}{\text{volume (m}^3\text{)}} \quad (\text{kg/m}^3)$$

We can use any units of mass and volume: g/cm^3, tonnes/km^3, …

Mass (kg)		
Volume (m^3)		1

To calculate density, we can use the formula or a ratio table. On a ratio table, the density appears when the volume is 1 (mass per unit volume).

A copper pipe has a mass of 450g.
It has a volume of 50cm^3.
What is its density?

$$\text{Density} = \frac{\text{mass (g)}}{\text{volume (cm}^3\text{)}} \quad (\text{g/cm}^3)$$

$$= \frac{450\text{ g}}{50\text{ cm}^3}$$

$$= 9\text{ g/cm}^3$$

÷ 50

Mass (g)	450	9
Volume (cm^3)	50	1

÷ 50

9 g/cm^3

Both representations show the same thing!

Working out Mass

A block of metal has a density of 8 g/cm^3 and volume of 56 cm^3.
Work out its mass.

× 56

Mass (g)		8
Volume (cm^3)	56	1

× 56

Using the ratio table, you can work out the mass is 448g.

Working out Volume

A block of metal has a density of 7.2 g/m^3 and mass of 1800g.
Work out its volume.

Mass (g)	1800	7.2
Volume (m^3)		1

÷ 7.2 ÷ 7.2

Remember you can work vertically as well as horizontally with ratio table.
The volume will be 250cm^3

Keyword/Skill	Definition/Tips
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Year 11 – Maths – Unit NP13 – Advanced Proportion & Rates of Change

Pressure

Pressure is a compound measure of force per unit area.

Its units are a combination of force and area.

$$\text{Pressure} = \frac{\text{force (N)}}{\text{area (m}^2\text{)}}$$

(N/m²)

To calculate pressure, we can use the formula or a ratio table.
On a ratio table, the pressure appears when the area is 1
(force per unit area).

$$\text{Pressure} = \frac{\text{force (N)}}{\text{area (m}^2\text{)}}$$

(N / m²)

Force (N)		
Area (m²)		1

To calculate pressure, we can use the formula or a ratio table.
I push with a force of 5 N over an area of 0.5 m².
What pressure do I apply?

$$\text{Pressure} = \frac{\text{force (N)}}{\text{area (m}^2\text{)}}$$

(N/m²)

$$= \frac{5}{0.5}$$

$$= 10 \text{ N/m}^2$$

Force (N)	5	10
Area (m²)	0.5	1

× 2 (above 5 → 10)

× 2 (below 0.5 → 1)

10 N/m²

Both representations show the same thing!

Working out Force

An object applies a pressure of 15 N/cm² to an area of 20 cm².
Work out the force.

Force (N)		15
Area (cm²)	20	1

× 20 (above 20 → 1)

× 20 (below 15 → 300)

Using the ratio table, you can work out the mass is 300N.

Working out Area

An object applies a force of 171 N with a pressure of 4.5N/m².
Work out the area.

Force (N)	171	4.5
Area (m²)		1

÷ 4.5 (above 4.5 → 1)

÷ 4.5 (below 171 → 38)

Remember you can work vertically as well as horizontally with ratio table.
The volume will be 38cm².

Keyword/Skill	Definition/Tips
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Year 11 – Maths – Unit NP13 – Advanced Proportion & Rates of Change

Speed

Speed is a compound measure of distance per unit time.

Its units are a combination of distance and time.

(miles per hour, m/s, m/min, km/h, miles/day, …)

$$\text{Speed (m/s)} = \frac{\text{distance (m)}}{\text{time (s)}}$$

To calculate speed, we can use a formula or a ratio table.
On a ratio table, the speed appears when the time is 1 (distance per unit time).

$$\text{Speed (m / s)} = \frac{\text{distance (m)}}{\text{time (s)}}$$

Distance (m)		
Time (s)		1

Keyword/Skill	Definition/Tips
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To calculate speed, we can use the formula or a ratio table.
A journey from Manchester to Birmingham is 100 miles over a time of 2 hours.
What is the average speed for the journey?

$$\text{Speed (miles/hour)} = \frac{\text{distance (m)}}{\text{time (h)}}$$

$$= \frac{100}{2}$$

= 50 miles per hour

		÷ 2	
Distance (m)	100	50	
Time (h)	2	1	50 mph
		÷ 2	

Both representations show the same thing!

Working out Distance

A bus has an average speed of 40km/h. It drives for 6 hours. What is the distance it drives?

		× 6	
Distance (km)		40	
Time (h)	6	1	
		× 6	

Using the ratio table, you can work out the distance is 240km.

Working out Time

A helicopter travels 800 miles and travels at an average speed of 160mph.
How long would this journey take

Distance (km)	800	160	÷ 160
Time (h)		1	
÷ 160			

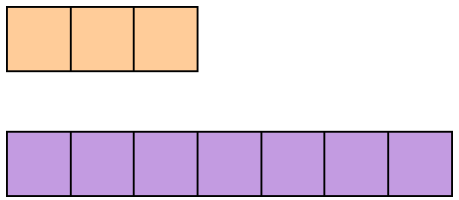
Remember you can work vertically as well as horizontally with ratio table.
The time will be 5 hours.

Year 11 – Maths – Unit NP13 – Advanced Proportion & Rates of Change

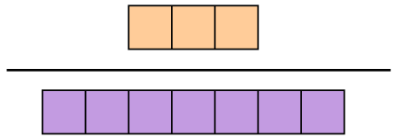
Ratio as Fractions

There are lots of fraction relationships hidden in a ratio.

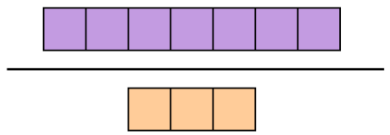
$a : b = 3 : 7$



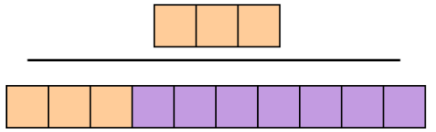
a is $\frac{3}{7}$ of b



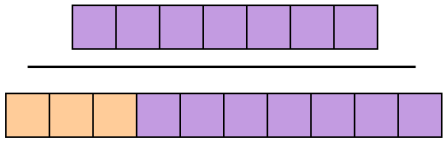
b is $\frac{7}{3}$ of a



a is $\frac{3}{10}$ of the total



b is $\frac{7}{10}$ of the total



Combining Ratios

If we want to combine ratios that seem not to 'match', we must scale them first.

Red : Yellow is 2 : 3.
Red : Blue is 6 : 7.

Write the ratio Red : Yellow : Blue

	R	Y	B
× 3	2	3	× 3
	6	9	7

$R : Y : B = 6 : 9 : 7$

Sometimes, you will need to scale both ratios to combine them.

Red : Yellow is 3:2.
Yellow : Blue is 3:7.

Write the ratio Red : Yellow : Blue
Yellow is the common one, we need a common multiple of 2 and 3.
We will use 6 (as it is the lowest common multiple).

	R	Y	B	
× 3	3	2		
× 2		3	× 3	7
	9	6		14

× 2

$R : Y : B = 9 : 6 : 14$