Year 10 Foundation Maths KO
Autumn Term 2026

Pages	Topic
2 – 3	Introduction to Algebraic Thinking
4 – 6	Manipulating and Simplifying Equations
7 – 8	Expanding & Factorising
9 – 10	Linear Equations
11 – 14	Formulae
15 – 16	Drawing, Measuring & Constructing
17 – 22	Angles & Polygons
23 – 25	Area
26 – 27	Place Value & Number Line
28 – 29	Addition & Subtraction
30 – 33	Multiplying & Dividing
34 – 35	Powers, Roots, Primes
36 – 37	Order of Operations
38 – 40	Directed Number
41 – 46	Fractions

Year 10 – Maths – Unit A1 – Introduction to Algebraic Thinking

This tile represents the number 1:

This tile represents any number:

We can think of it as representing an *unknown number that can change its value*.
We call this a variable number or variable for short.

When we hold the tile, or work with it on-screen, it won't change size, but we know that its value can change.

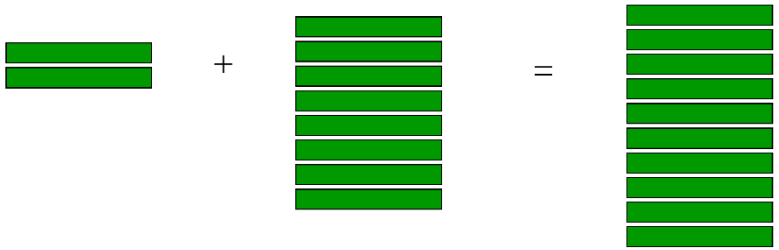
My mathematical journey

What do I need to remember from before?	What will I learn about in this unit?	Where does this lead?
Directed numbers (NP6)	Variable unknowns	Simplifying expressions (A2)
	Algebraic expressions	Multiplying expressions (A3)
	Substitution	Linear equations (A4)
	Equations	Formulae (A5)

Generalising with tiles

The variable tile helps us to picture that

2 lots of a number + 8 lots of the same number = 10 lots of that number



2 + 8 = 10

Generalising with symbols

2 lots of a number + 8 lots of the same number = 10 lots of that number

2 · number + 8 · number = 10 · number

This is an awkward, long sentence to write. Let's shorten it.

$2 \cdot n + 8 \cdot n = 10 \cdot n$

Let's write n instead of 'number'.

$2n + 8n = 10n$

Let's not worry about writing the multiplication symbol.

2 + 8 = 10

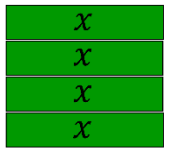
Key information

Remember, the letters stand for *unknown variable numbers* – numbers that we don't know and that can change their value.

x

These unknown variable numbers are called variables.
Any known, fixed numbers are called constants.

constant → $4x$ ← variable



So now we can say that an expression contains variables, constants and operations.

constant → $4x$ ← variable

operation of multiplication

Remember, $4x$ means "4 lots of a number", so we could write $4 \times x$ or $4 \cdot x$ but we don't write any symbol.



Everyone in mathematics knows not to write the multiplication symbol when working in algebra, so it doesn't get confusing.

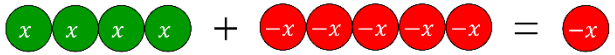
Keyword/ Skill	Definition/Tips
Variable	A number that can change its value, represented by a letter such as x or a green tile when we do not know its value.
Constant	A number that does not change, is fixed.
Operation	Something that takes input numbers and turns them into output numbers, such as =, -, x, and ÷, exponentiation and roots.
Term	The parts of an expression separated by + or -.
Expression	A mathematical statement written using symbols, numbers or letters.
Simplifying	Collect 'like terms'.
Substitution	In algebra it means replacing letters with numbers.
Equation	A statement showing that two expressions are equal.

Year 10 – Maths – Unit A1 – Introduction to Algebraic Thinking

Simplifying Expressions:

Sometimes it's easier to draw circle tiles.
Here how our last example looks with circle tiles.

$$4x - 5x = -x$$

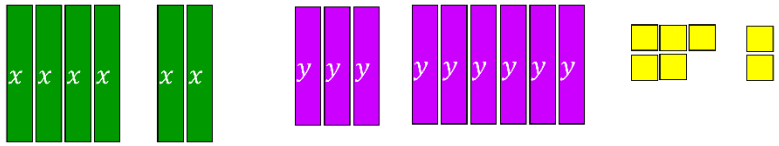


Notice how whenever we have one x we just write x, never $1x$.
This is the same with negatives. We don't write $-1x$, just $-x$.

Like terms:

To simplify expressions, we collect the like terms and simplify them.

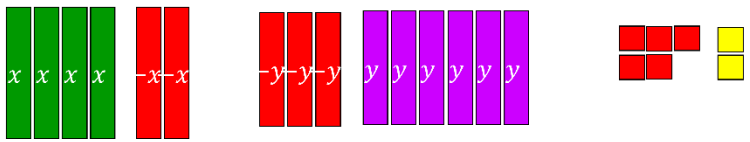
$$4x + 3y + 2x + 5 + 6y + 2$$



$$= 6x + 9y + 7$$

We always represent the negative of a number or variable with a red tile.
This means we have to remember to identify which terms are like terms without thinking about colour because all negative terms will be red.

$$4x - 3y - 2x - 5 + 6y + 2$$



$$= 2x + 3y - 3$$

Substitution:

When we substitute a number for x, we can picture the expression, then assign x that number.

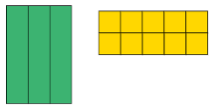
$2x + 5$ 

When $x = 4$, $2x + 5$ looks like this  and its value is 13.

When $x = 10$, $2x + 5$ looks like this  and its value is 25.

Let's evaluate $3x + 10$ when $x = 4$.

We have two ways of writing the working out.



$3x + 10$ when $x = 4$

$= 3 \cdot 4 + 10$
$= 12 + 10$
$= 22$

$= 3(4) + 10$
$= 12 + 10$
$= 22$

Let's evaluate $3x + 10$ when $x = -4$.

$3x + 10$ when $x = -4$

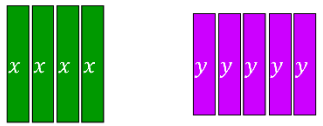
$= 3 \cdot -4 + 10$
$= 12 + 10$
$= 22$

$= 3(-4) + 10$
$= 12 + 10$
$= 22$

When x is substituted for a negative number, it's clearer to write a bracket around it.

When we have an expression with more than one variable, we can substitute different numbers for each.

$$4x + 5y$$



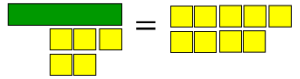
When $x = 2$ and $y = 1$, $4x + 5y = 4 \cdot 2 + 5 \cdot 1 = 13$
When $x = 8$ and $y = 10$, $4x + 5y = 4 \cdot 8 + 5 \cdot 10 = 82$
When $x = 5$ and $y = 5$, $4x + 5y = 4 \cdot 5 + 5 \cdot 5 = 45$

Solving Equations:

If we have an equation like

$$x + 5 = 9$$

we can work out what value of x makes it true. In this case $x = 4$.
We call this solving the equation. We say $x = 4$ is the solution to the equation.

$x + 5 = 9$ 

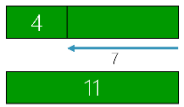
$$x + 5 = 4 + 5$$

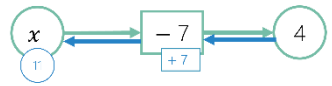
Comparing both sides of the equals sign, we see that $x = 4$ for this equation.

Some equations can be solved by thinking about fact families and inverse operations. We can use function machines to help us.
Here is an equation

$$x - 7 = 4$$

We can read it as "Take an unknown number, subtract 7, and we reach 4."





Inverse operations tell us to start at 4, add 7, and we'll reach x.

Every time we solve an equation, we double-check the solution by substituting it for x.

Our equation was $x - 7 = 4$.
We said the solution was $x = 11$.

We check by substituting 11 for x:

$$x - 7 = 4$$
$$11 - 7 = 4 \quad \checkmark$$

This quick check guarantees we've got the correct solution.

Year 10 – Maths – Unit A2 – Manipulating and simplifying expressions 1

This tile represents the number 1:

This tile represents any number:

We can think of it as representing an *unknown number that can change its value*.
We call this a variable number or variable for short.

When we hold the tile, or work with it on-screen, it won't change size, but we know that its value can change.

My mathematical journey

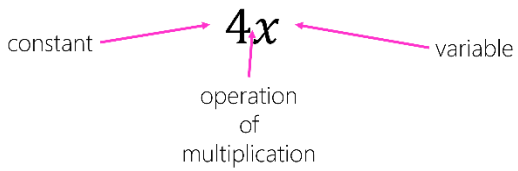
What do I need to remember from before?
- Exponents (NP4)
- Directed numbers (NP6)
- Expressions (A1)

What will I learn about in this unit?
- Adding and subtracting expressions
- Multiplying and dividing expressions
- Index laws
- Forming expressions

Where does this lead?
- Expanding and factorising brackets (A3)
- Solving equations (A4)
- Quadratic expressions (A11)

Information to recall from unit A1

We learnt about expressions, which contain variables, constants and operations.



Remember, $4x$ means "4 lots of a number", so we could write $4 \times x$ or $4 \cdot x$ but we don't write any symbol.

Everyone in mathematics knows not to write the multiplication symbol when working in algebra, so it doesn't get confusing.

We learnt about terms.

$6x + 8y$

These are terms.

They are separated by the + operator.

$\frac{a}{2} - 3$

These are terms.

They are separated by the − operator.

$6x + 8y$

These are both positive terms.

The terms are $6x$ and $8y$.

We learnt about like terms and unlike terms.

$4x + 3y + 2x + 5 + 6y + 2$

is like
is like
is like

is unlike
is unlike
is unlike

$\frac{a}{2} - 3$

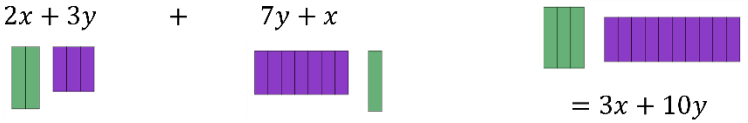
$\frac{a}{2}$ is a positive term.

3 is a negative term, because subtracting 3 is the same as adding $^-3$.

The terms are $\frac{a}{2}$ and -3.

Adding and subtracting expressions

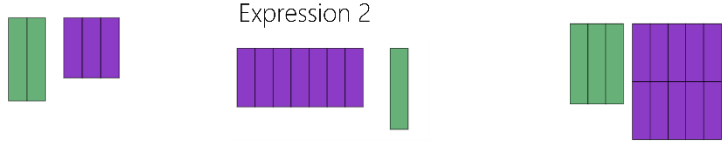
We can add expressions.



If you see a bracket around an expression, it can help you to think of it separately to everything else.

$$(2x + 3y) + (7y + x) = 3x + 10y$$

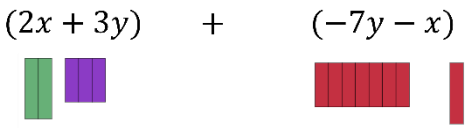
Expression 1 + Expression 2 = their sum



We can subtract expressions.



Remember that subtraction is the same as adding the inverse.



Keyword/ Skill	Definition/Tips
Variable	A number that can change its value, represented by a letter such as x or a green tile when we do not know its value.
Constant	A number that does not change, is fixed.
Operation	Something that takes input numbers and turns them into output numbers, such as +,-,x and ÷
Expression	A collection of constants, variables and operations.
Term	The parts of an expression separated by + or -.
Exponent	Is a count of how many variables to multiply together. Usually, they are written as x^n.

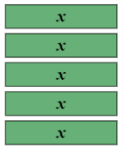
Year 10 – Maths – Unit A2 – Manipulating and simplifying expressions 1

Multiplying expressions:

We can multiply variables by constants.

$5 \times x$

$5 \cdot x$

5 lots of x →  → $5x$

$$5 \times x = 5x$$

Multiplication is commutative.

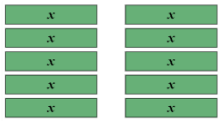
$$x \times 5 = 5 \times x$$

$$5x$$

We always write the constant before the variable.

$5 \times x \times 2$

$= 10x$



$3y \times 4$

$= 12y$

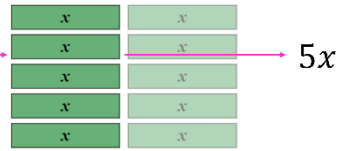


Dividing Expressions:

We can divide variables by constants.

$10x \div 2$

$\frac{10x}{2}$

Half of $10x$ →  → $5x$

$$\frac{10x}{2} = 5x$$

Repeated multiplication and index notation (exponents):

We learnt in NP4 that we can use index notation to write repeated multiplication. $2 \times 2 \times 2 \times 2 \times 2 = 2^5$

We can also use index notation with variables. $x \cdot x \cdot x \cdot x \cdot x = x^5$

Multiplication index law

$x^2 \times x^3 = x^5$ means we have a total count of 5 xs multiplied together.

$y^4 \times y^5 = y^9$ means we have a total count of 9 xs multiplied together.

When we multiply powers with the same base, we can add their exponents.

Repeated multiplication

$(x^3)^6 = x^{18}$ means we had a count of three xs multiplied together and we repeated six times, to give a count of eighteen xs multiplied together.

$(y^4)^3 = y^{12}$ means we had a count of four ys multiplied together and we repeated three times, to give a count of twelve ys multiplied together.

When we find a power of a power, we can multiply the exponents together.

Division index law

$\frac{x^3}{x^2} = x$ means we had a count of three xs multiplied together, but we 'undid' two of them, leaving only one.

$\frac{y^7}{y^4} = y^3$ means we had a count of seven ys multiplied together, but we 'undid' four of them, leaving three.

When we divide powers with the same base, we can subtract their exponents.

Simplifying expressions with more complex terms

Now we can multiply two expressions together.

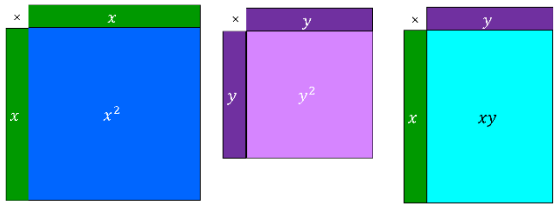
$2x \times 3y = 2 \cdot x \cdot 3 \cdot y = 6xy$

$2x \times 3x = 2 \cdot x \cdot 3 \cdot x = 6x^2$

$2x \times 3x \times y = 2 \cdot x \cdot 3 \cdot x \cdot y = 6x^2y$

Simplifying more complex expressions:

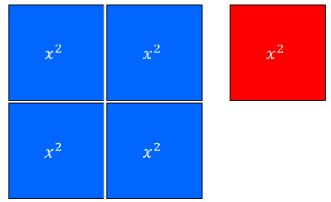
We can use algebra tiles to see other expressions.



All of these expressions, x^2, y^2 and xy are unlike and cannot be collected.

Zero pairs still work.

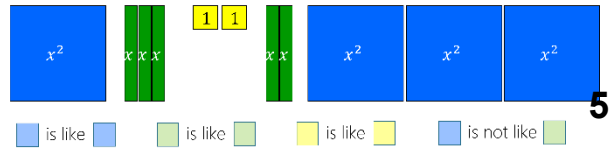
$4x^2 - x^2 = 3x^2$



We can use the x^2 tile to help us simplify more complex expressions.

We know that we can only add together like terms.

$x^2 + 3x + 2 + 2x + 3x^2 = 4x^2 + 5x + 2$



Forming expressions:

Whenever we do not know the value of a number, we can write a letter in its place.

We call this letter an unknown.

Writing unknowns works in the same way as writing numbers.

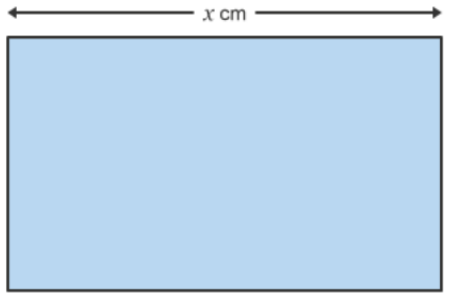
Example 1

Pens are sold in packs of 6 and rulers are sold in boxes of 10.
A teacher buys *p* packs of pens and *r* boxes of rulers.
Write an expression for the total number of pens and rulers bought.

There are 6 pens in each pack, so the number of pens bought is $6\times p$ which is $6p$.
There are 10 rulers in each box, so the number of rulers bought is $10\times r$ which is $10r$.
The number of pens and rulers bought is $6p + 10r$

Example 2

A rectangle has a width of x cm.
The height is 3 cm less than the width.
Write an expression for the perimeter of the rectangle.



The perimeter is found by adding together the lengths of the sides of a shape.
The width of the rectangle is given as x cm.
The height of the rectangle is 3 less than the width: $x-3$ cm
Perimeter = $x+x+(x-3)+(x-3)$
Perimeter = $(4x-6)$ cm

Example 3

Abi is x years old.
Bob is twice as old as Abi.
Cam is 3 years older than Bob.
Form an expression for Bob and Cam's ages in terms of x.

Bob → $2 \times x$
$2x$

Cam → Bob $+3$
$2x + 3$

Year 10 – Maths - Unit A3 – Expanding & Factorising

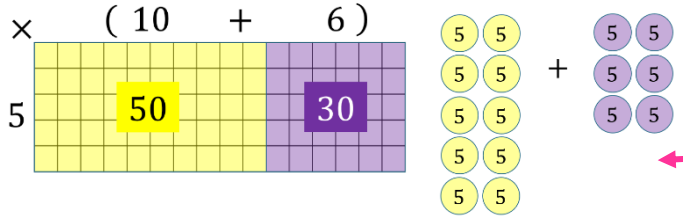
My mathematical journey

What do I need to remember from before?	What will I learn about in this unit?	Where does this lead?
Area models for multiplication (NP3)	Expanding expressions with brackets	Solving equations (A4)
Collecting like terms (A2)	Factorising expressions as the opposite of expanding	Formulae (A5)
	Expanding two brackets	Inequalities (A8)
		Quadratic expressions (A11)

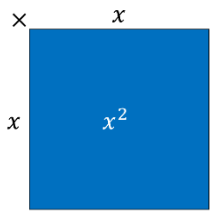
Distributive Property

$5 \times 16 = 5 \times (10 + 6) = 5 \times 10 + 5 \times 6$

We have learnt about the distributive property in NP3.



We can use the distributive property with **constants** and with **variables**.



When terms are multiplied, they become squared. We show this with a power/index of 2:

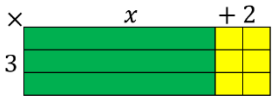
$$x \times x = x^2$$

Expanding brackets

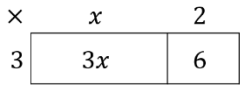
We can use the **distributive property** to expand brackets.

Here is $3(x + 2) = 3x + 6$.

Tiles

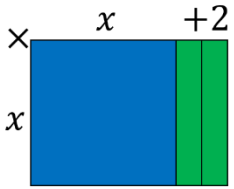


Area model

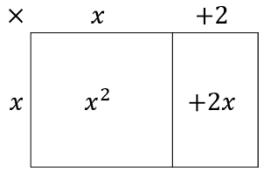


×	x	2
3	$3x$	6

Tiles

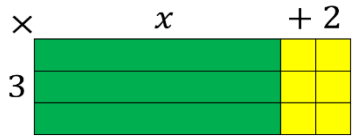


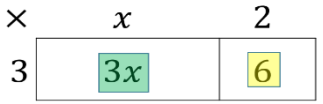
Area model



×	x	2
x	x^2	$2x$

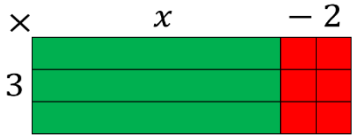
We see that $x(x + 2) = x^2 + 2x$

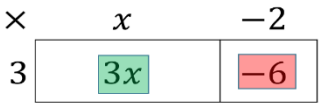




Factorised form: $3(x + 2) = 3x + 6$ Expanded form.

Factorising is the **inverse** of expanding





Factorised form: $3(x - 2) = 3x - 6$ Expanded form.

Keyword/ Skill	**Definition/Tips**
Variable	A number we do not know yet, it is usually represented by a letter or a green tile.
Constant	A fixed value, a number that does not change.
Term	Either a single number or a **variable,** such as 4 or n or 3a or 6y.
Expression	A mathematical statement written using **symbols**, **numbers** or **letters**.
Simplifying Expressions	**Collecting 'like terms'.** Be careful with negatives. x^2 and x are not like terms.
Expand	When we **multiply** a **term** across a bracket, e.g. 3(a + 2) = 3a + 6
Factorise	The **inverse** of **expand**. When we divide an **expression** by all **common factors** or **terms**, e.g. 6g + 4 = 2(3g + 2) and $a^2 - 2a = a(a - 2)$

Year 10 – Maths - Unit A3 – Expanding & Factorising

Expanding 2 sets of single brackets

We can expand 2 sets of brackets and then simplify the result.

Remember: subtraction is adding the inverse, so $x - 1$ is multiplied by 3 *and* flipped so that we can add it in.

$$2(x + 3) - 3(x - 1)$$



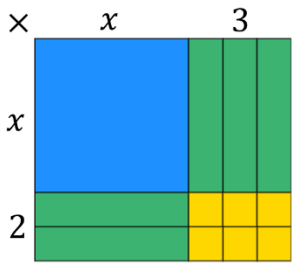
Be very careful when there is a subtraction between multiplications.

We have $2x + 6 - 3x + 3$ which simplifies to $9 - x$

Expanding double brackets

We can expand double brackets and then simplify the result.

$$(x + 2)(x + 3)$$

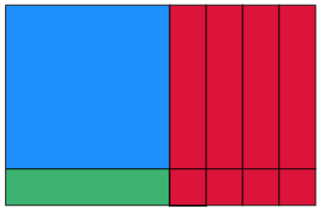


×	x	3
x		
2		

Factorised form (multiplication)

$$(x + 1)(x - 4)$$

Tiles



Area model/grid

×	x	-4
x	x^2	$-4x$
1	x	-4

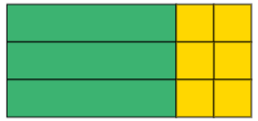
Expanded form (sum)

$$x^2 - 3x - 4$$

Factorising

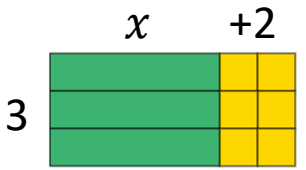
If we have an expression arranged in a rectangle, we can see what multiplication might have produced it.

Here is the expression $3x + 6$ in tiles and an area model.



×	$3x$	6

We can see that the expression $x + 2$ has been multiplied by the constant 3.



×	x	+2
3	$3x$	6

The factorised form of $3x + 6$ is $3(x + 2)$

$4x + 8 = 1(4x + 8)$



Not very interesting.
We can do this with any expression.

$4x + 8 = 2(2x + 4)$



Factorised.
We have found that $4x$ and 8 have a *common factor* of 2.

$4x + 8 = 4(x + 2)$

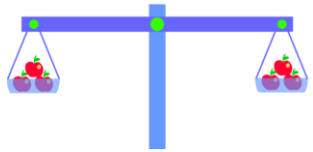


Fully factorised.
We have found the *highest common factor* of $4x$ and 8, which is 4.

Year 10 – Maths - Unit A4 – Linear Equations

Solving Equations

When we are solving equations, we need to figure out the value of the variable (usually a letter) in the equation.



Variable

$$4x - 7 = 5$$

Equations work like a weighing scale; it must always be balanced/equal. If I add or remove something from one side of the equals, I need to do the same to the other side to keep it balanced/equal.

Balancing Equations

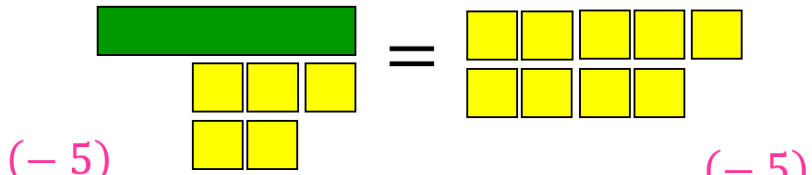
This equation has remained balanced, because both sides of the equals have been changed in the same way each time.

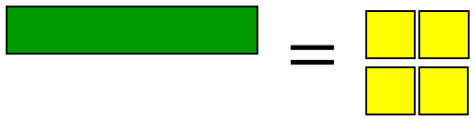
$$6 + 2 = 9 - 1$$

$(+1)$ $\quad 6 + 2 + 1 = 9 - 1 + 1 \quad (+1)$

$(\div 2)$ $\quad \frac{6+2+1}{2} = \frac{9-1+1}{2} \quad (\div 2)$

We can do the same with pictorial representations of equations





And using algebraic symbols

$$x + 5 = 9$$

(-5) $\qquad$ (-5)

$$x = 4$$

My mathematical journey

What do I need to remember from before?

Equality & inverse operations (NP2, NP3, NP4)

Solving equations (A1)

Simplifying expressions (A2)

Expanding brackets (A3)

What will I learn about in this unit?

Mathematical equality

Balancing an equation

Solving all types of linear equations

Where does this lead?

Rearranging formulae (A5)

Equations of a line (A6)

Quadratic equations (A12)

Using equations to solve geometry and probability problems (GM2 – GM11, SP7)

Keyword/Skill	**Definition/Tips**
Variable	A symbol for a number we do not know yet, it is usually a letter.
Term	Either a single number or a **variable**, such as 4 or n or 3a or 6y.
Expression	A mathematical statement written using **symbols**, **numbers** or **letters**.
Equation	A statement showing that **two expressions are equal.**
Formula	Shows the **relationship** between **two or more variables.**
Simplifying Expressions	**Collect 'like terms'.** Be careful with negatives. x^2 and x are not like terms.
Expand	When we **multiply** a **term** across a bracket, e.g. 3(a + 2) = 3a + 6

Maths - Unit A4 – Linear Equations

Solving Equations

We can sometimes solve some equations using our knowledge of number facts and calculations.

E.g. $3 + x = 10$

What would you need to add to 3 to get 10?

$x = 7$

E.g. $3y = 15$

What number, when multiplied by 3 equals 15?

$y = 5$

E.g. $z - 9 = 20$

What number, when you subtract 9 from it equals 20?

$z = 29$

However, with more complex equations, we need to use **balancing** to solve them.

e.g. $2x + 12 = 28$

-12 | -12

$2x = 16$

÷2 | ÷2

$x = 8$

We identify the operations that can be applied to both sides of the equals to get the unknown value on its own.

e.g $10x - 5 = 33$

+5 | +5

$10x = 38$

÷10 | ÷10

$x = 3.8$

Unknown values don't have to be integers.

Unknowns on both sides

$3x - 4 = 7x + 7$

$-3x$ | $-3x$

$-4 = 4x + 7$

-7 | -7

$-11 = 4x$

÷4 | ÷4

$-\frac{11}{4} = x$

Solving Equations with fractions and brackets

If your unknown value is the denominator of a fraction, your first step to maintain balance is to multiply by the denominator.

$\frac{12}{y} = 2$

× y | × y

$12 = 2y$

÷2 | ÷2

$6 = y$

If there are brackets, expand them first

$2(x + 4) = 2$

$2x + 8 = 2$

Then continue to solve using balancing.

Keyword/Skill	Definition/Tips
Variable	A symbol for a number we do not know yet, it is usually a letter.
Term	Either a single number or a **variable**, such as 4 or n or 3a or 6y.
Expression	A mathematical statement written using **symbols**, **numbers** or **letters**.
Equation	A statement showing that **two expressions are equal.**
Formula	Shows the **relationship** between **two or more variables.**
Expand	When we **multiply** a **term** across a bracket, e.g. 3(a + 2) = 3a + 6
Simplifying Expressions	**Collect 'like terms'.** Be careful with negatives. x^2 and x are not like terms.

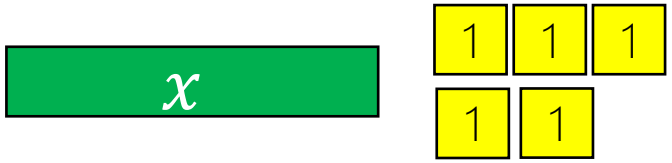
Year 10 – Maths – Unit A5 – Formulae

Substitution

constant → $4x$ ← variable

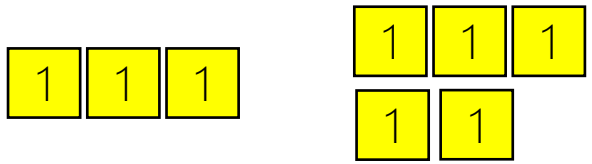
Here is an expression.

$$x + 5$$



We can substitute 3 for x $(x = 3)$

$$3 + 5$$

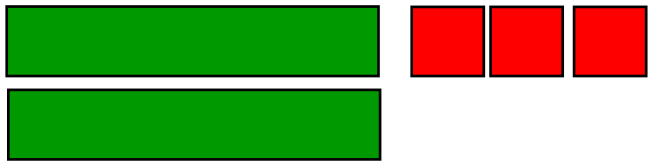


The expression will have a different value based on the number we substitute in!

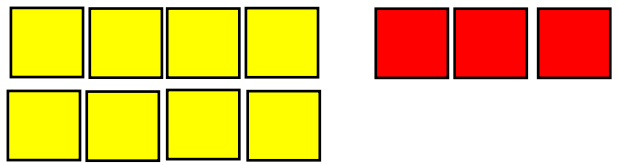
We can now evaluate the expression: 3 + 5 = 8

Here is another expression.

$$2x - 3$$

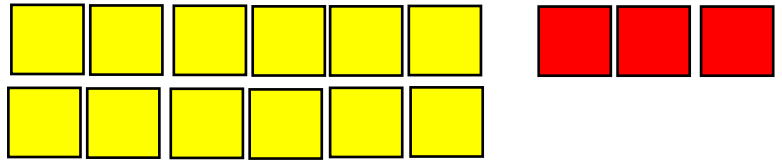


When $x = 4$



8 – 3 = 5 the expression has a value of 5.

When $x = 6$



12– 3 = 9 the expression has a value of 9.

My mathematical journey

What do I need to remember from before?

- Arithmetic strategies (NP1, 2, 3, 4)
- Order of operations (NP5)
- Negative numbers (NP6)
- Algebraic expressions (A1, 2, 3)
- Solving equations (A4)

What will I learn about in this unit?

- Substituting numbers into expressions and formulae
- Writing and using formulae
- Rearranging formulae to change the subject

Where does this lead?

- All further algebra units
- Using formulae in geometry (GM3 onwards)
- Advanced proportion (NP13)
- Advanced probability and statistics (A Level)

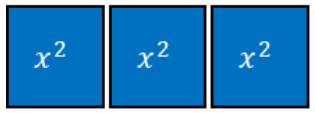
Keyword/Skill	Definition/Tips
Variable	A letter that represents any number (a letter whose value can vary). 3.g. e, y
Constant	A fixed number
Expression	A collection of any variables,, constants and operations
Substitute	Replace a variable with a constant.
Evaluate	Work out the value of a calculation
Formula/Formula e	A set of instructions to work something out. A formula can be written in words, as an expression or as an equation.
Subject	The subject of a formula is the variable that is 'on its own' on one side of the equation. e.g. in the formula $A = lw$, A id the subject.
Rearrange	Rearranging a formula means changing its subject.

Year 10 – Maths – Unit A5 – Formulae

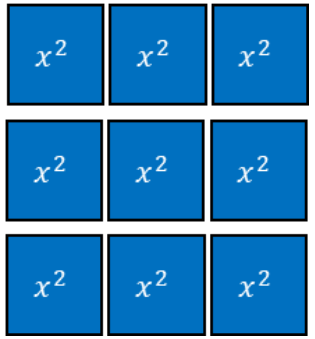
Quadratic Substitution

Look at the difference between the expressions $3x^2$ and $(3x)^2$.

$3x^2$



$(3x)^2$



If $x = 2$

$3x^2 = 3(2)^2 = 12$

$(3x)^2 = \big(3(2)\big)^2 = (6)^2$

Make sure you read the question carefully and apply the order of operations.

Example:

When $a = 2$ and $b = 5$, evaluate:

$(a + b)^2$ and $a + b^2$

$(a + b)^2$

$= (2 + 5)^2$

$= (7)^2$

$= 49$

$a + b^2$

$= 2 + (5)^2$

$= 2 + 25$

$= 27$

Formula

A formula is a set of instructions for working something out.

Here is a formula written in words.

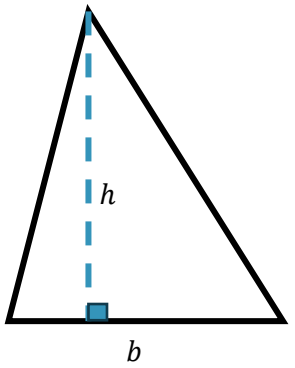
half base × height

Here is the same formula, written using numbers and letters.

$\frac{1}{2}bh$

Here is the same formula, written as an equation.

$A = \frac{1}{2}bh$



We can substitute into this formulae.

If the base was 10cm and the height was 6cm:

$A = \frac{1}{2} \times 10 \times 6 = 30cm^2$

Keyword/Skill	Definition/Tips
Variable	A letter that represents any number (a letter whose value can vary). 3.g. e, y
Constant	A fixed number
Expression	A collection of any variables,, constants and operations
Substitute	Replace a variable with a constant.
Evaluate	Work out the value of a calculation
Formula/ Formulae	A set of instructions to work something out. A formula can be written in words, as an expression or as an equation.
Subject	The subject of a formula is the variable that is 'on its own' on one side of the equation. e.g. in the formula $A = lw$, A id the subject.
Rearrange	Rearranging a formula means changing its subject.

Year 10 – Maths – Unit A5 – Formulae

Writing Formulas

We can make formulas in lots of different contexts.

Example:

A taxi journey costs £3.50 plus £2 for every mile.

We can represent this as a formula.

$C = 3.5 + 2m$

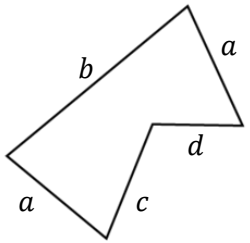
Where C is the cost and m is the number of miles.

You can now find the cost of any journey.
If the journey was 5 miles:

$$C = 3.5 + 2(5) = 13.5 = £13.50$$

Example:

Write a formula for the perimeter of this shape.
Use P for perimeter.



Perimeter is adding up the distance around the outside.

$$P = a + b + a + c + d$$

Simplify if you can:

$$P = 2a + b + c + d$$

Forming and Solving Equations

We can work information out by forming an equation and then solving it.

Here is a rectangle:

4cm 

Its perimeter is 34cm.

We can find its length by setting up an equation. In these types of questions, you need to:

- Give the unknown a letter.
- Write a formula/form an equation.
- Substitute in what you know.
- Solve to find the unknown.

Here we don't know the length so I will call that x.

x cm

4cm 

$$P = 4 + x + 4 + x = 2x + 8$$
$$P = 2x + 8$$

The perimeter is 34cm so we can substitute that in and solve the equation.

$$34 = 2x + 8$$
$$26 = 2x$$
$$13 = x$$

The length of the rectangle is 13cm.

Keyword/Skill	Definition/Tips
Variable	A letter that represents any number (a letter whose value can vary). 3.g. e, y
Constant	A fixed number
Expression	A collection of any variables,, constants and operations
Substitute	Replace a variable with a constant.
Evaluate	Work out the value of a calculation
Formula/ Formulae	A set of instructions to work something out. A formula can be written in words, as an expression or as an equation.
Subject	The subject of a formula is the variable that is 'on its own' on one side of the equation. e.g. in the formula $A = lw$, A id the subject.
Rearrange	Rearranging a formula means changing its subject.

Year 10 – Maths – Unit A5 – Formulae

Subject of the Formula & Rearranging Formula

The subject of a formula is the quantity the formula calculates.
If we write the formula as an equation, the subject appears on its own on one side of the equals sign.



In this formula, the area is the subject (A is the subject).

Here is a formula whose subject is a.

$$a = b + c$$

Because the formula is written as an equation, we can alter it by doing the same thing to both sides.

$$a = b + c$$

$$(-b) \quad (-b)$$

$$a - b = c$$

ow c is the subject.

We have changed the subject by rearranging the formula.

Here is a formula where y is the formula.

$$y = 2x + a$$

I want to make x the subject, so I want that on its own.

$$y = 2x + a$$

$$(-a) \quad (-a)$$

$$y - a = 2x$$

$$(\div 2) \quad (\div 2)$$

$$\frac{y - a}{2} = x$$

When dividing you can write it in the form of a fraction.

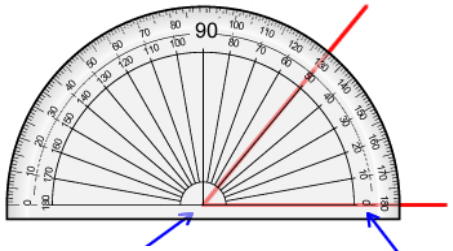
Keyword/Skill	Definition/Tips
Variable	A letter that represents any number (a letter whose value can vary). 3.g. e, y
Constant	A fixed number
Expression	A collection of any variables,, constants and operations
Substitute	Replace a variable with a constant.
Evaluate	Work out the value of a calculation
Formula/ Formulae	A set of instructions to work something out. A formula can be written in words, as an expression or as an equation.
Subject	The subject of a formula is the variable that is 'on its own' on one side of the equation. e.g. in the formula $A = lw$, A id the subject.
Rearrange	Rearranging a formula means changing its subject.

Year 10 – Maths – Unit GM1 – Drawing, Measuring & Constructing

Measuring an angle

- Accurate
- **Use** a protractor
- Use **pencil** and a ruler

Line up the bottom of the protractor with bottom line of the angle.



Make sure the vertex (corner) of the angle is lined up with the center of the protractor.

Start at zero and read up.

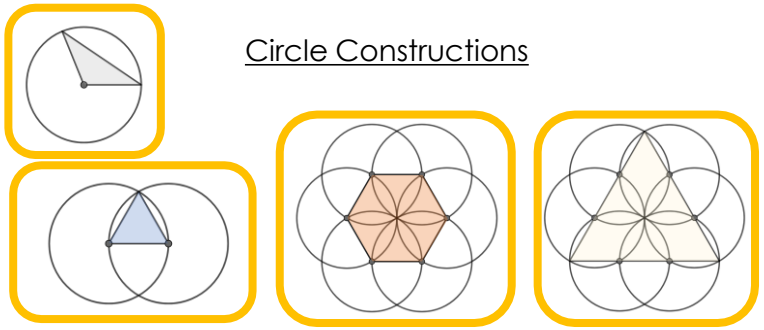
Estimating an Angle

- Is an educated guess!
- **Do not use** a protractor
- Consider the **angle properties**, is it acute (therefore smaller than 90°) or obtuse (bigger than 90° but smaller than 180°).

My mathematical journey

What do I need to remember from before?	What will I learn about in this unit?	Where does this lead?
Lines and angles (KS2)	Labelling lines and angles	Polygons and angles (GM2)
Measuring (KS2)	Drawing and measuring lines and angles	Congruence and similarity (GM4)
	Using compasses and a protractor	Advanced drawing, measuring and constructing (GM7)
	Constructions and loci	

Circle Constructions



By drawing circles, you can **construct** various **polygons**.

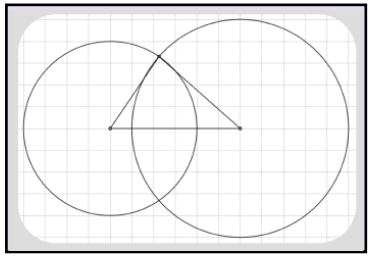
Acute	Right	Obtuse
$0° < \theta < 90°$	$90° = \theta$	$90° < \theta < 180°$

Straight	Reflex	Full turn
$180° = \theta$	$180° < \theta < 360°$	$360° = \theta$

Constructing Triangles – given 3 side lengths

Using **ruler** and **compasses**, you can **construct** any triangle, given its three side lengths.

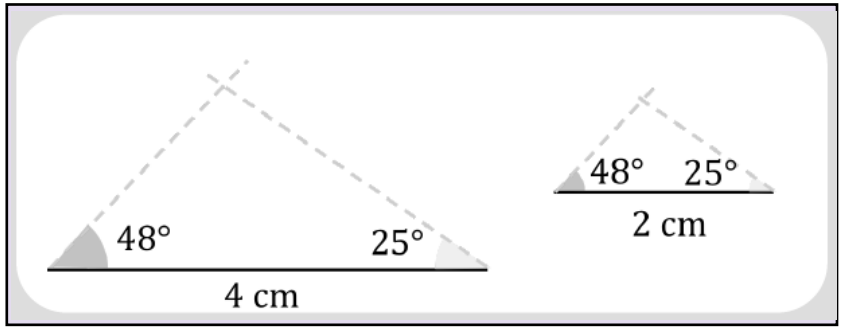
This triangle has side lengths 6cm, 5cm and 4 cm.
The 6cm line was drawn with a ruler. Then circles with radii 5cm and 4cm were constructed at either end of the 6cm line.
The intersection points show where the other vertex should be.



Using **ruler** and **protractor**, you can **construct** any triangle, given two of its angles.

These triangles both have interior angles of 48° and 25° but the side lengths are different.
Draw the side in between the two angles first; sometimes this is given to you in the question. Then measure the angles from each end of the line – make sure the protractor is lined up correctly! Extend your construction lines until they intersect (but don't rub them out – the examiner will want to see them).
The intersection points show where the other vertex should be.

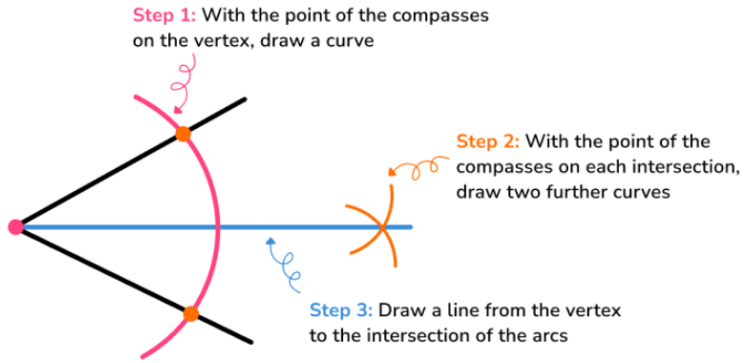
Constructing Triangles – given 2 angles



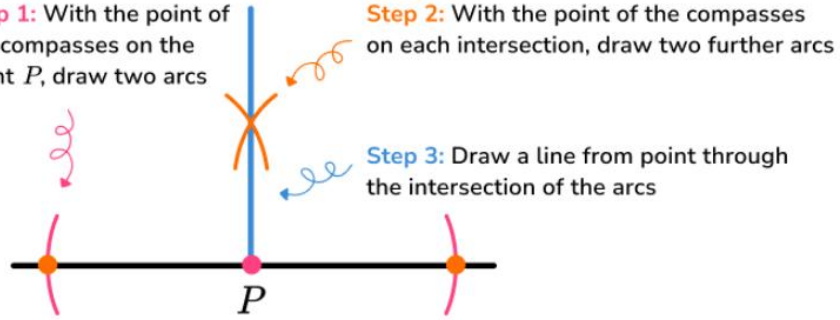
Year 10 – Maths – Unit GM1 – Drawing, Measuring & Constructing

Constructions

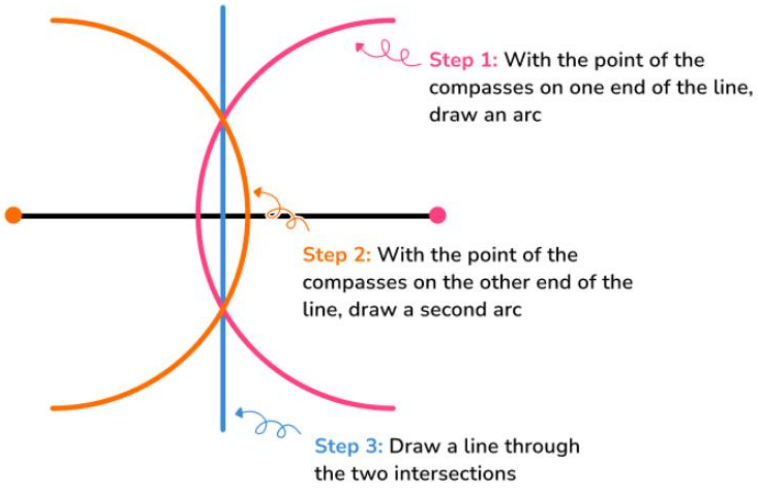
Angle bisector



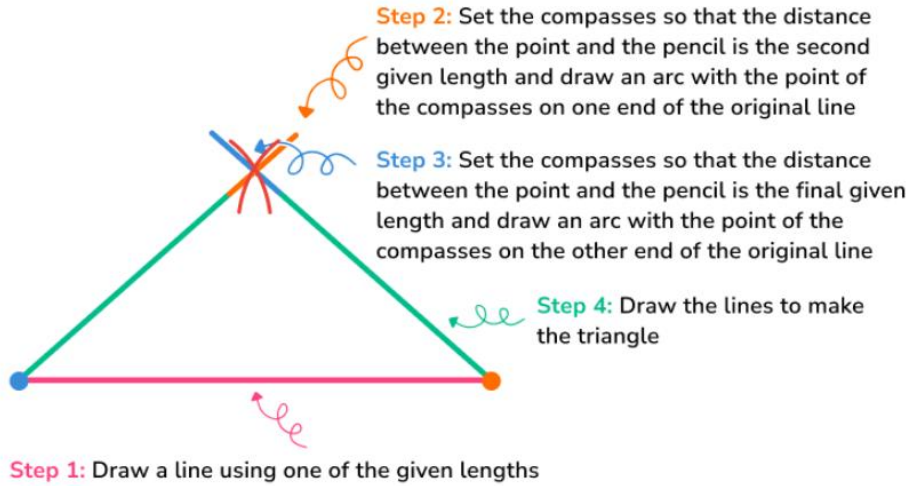
Perpendicular from a point on a line



Perpendicular bisector



Triangle given all three sides



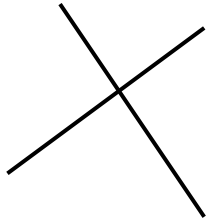
Keyword/Skill	Definition/Tips
Mensuration	The act or process of measuring
Pair of Compasses	A mathematical instrument that is used to draw circles and **arcs**
Point	A point has no length or width (it exists in no dimensions, or 0D)
Line	A line has infinite length ad no width (it exists in one dimension, or 1D) We use arrows to show its infinity in both directions.
Ray	A ray is a section of a line with a starting point that continues infinitely in one direction. We use an arrow to show its infinity in one direction.
Line segment	A line segment is a section of a line with a starting point and an end point
Construct	We construct when we only use compasses and a straight edge (ruler)
Bisector	Bisect means to cut in half, in to two equal pieces.
Perpendicular	Lines that meet at a right angle.
Equidistant	Equal distance from two points or lines.
Locus (plural – loci)	The path of all points that fit a condition

Year 10 – Maths – Unit GM2 – Angles & Polygons

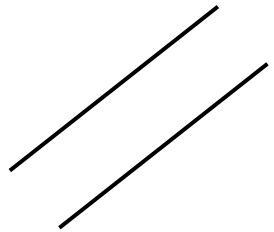


Horizontal

Vertical



Perpendicular – cross at right angles



Parallel

My mathematical journey

What do I need to remember from before?

Measuring and drawing angles (Key Stage 2, GM1)

Basic angle facts (NP2)

What will I learn about in this unit?

Angle facts about lines and polygons

Types of quadrilaterals and other polygons

Bearings

Where does this lead?

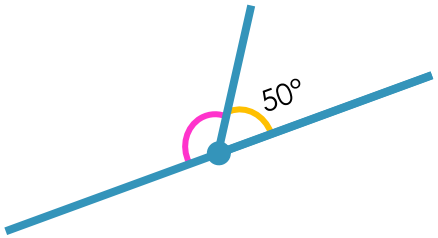
Congruence and similarity (GM4)

Trigonometry (GM5, GM9)

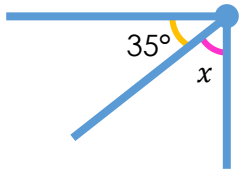
Solving geometric problems, including circle theorems (GM6, GM7, GM11)

Acute	Right	Obtuse	Straight	Reflex	Full turn
$0° < \theta < 90°$	$90° = \theta$	$90° < \theta < 180°$	$180° = \theta$	$180° < \theta < 360°$	$360° = \theta$

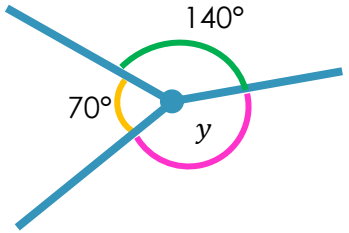
We can use some these facts to find missing angles.



Angles on a straight line sum to $180°$.
One of the angles is $50°$.
$180° - 50° - 130°$



A right angle is $90°$.
One of the angles is $35°$.
$90° - 35° = 55°$

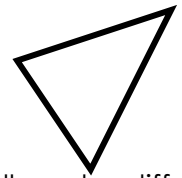


Angles around a point (full turn) is $360°$.
Two of the angles $70°$ and $140°$.
$360° - 70° - 140° = 150°$

Keyword/Skill	**Definition/Tips**
Angle	The amount of turn between two rays called arms meeting at a common point called vertex.
Vertically opposite	Pair of angles directly opposite to each other, formed by intersection of straight lines.
Reflex	Any angle that measures more than 180 degrees but less than 360 degrees.
Parallel	Equidistant lines, that is, exactly the same distance apart and never touching.
Perpendicular	Meeting or crossing at a right angle.
Vertices	A vertex (plural is vertices) is made when two lines meet.
Polygon	Two-dimensional closed shapes with straight sides are called polygons.
Acute	Any angle that measures less than 90 degrees.
Obtuse	Any angle that measures between 90 degrees and 180 degrees.
Right angle	Any angle that measures exactly 90 degrees.

Year 10 – Maths – Unit GM2 – Angles & Polygons

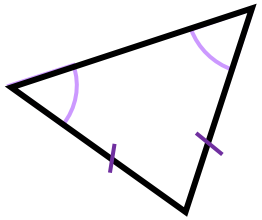
Scalene



All angles different

All sides a different length

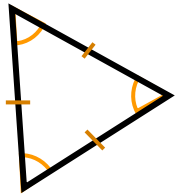
Isosceles



Two angles equal

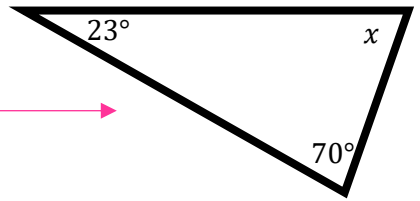
Two sides equal

Equilateral



All angles equal
Each angle is 60°
All sides equal

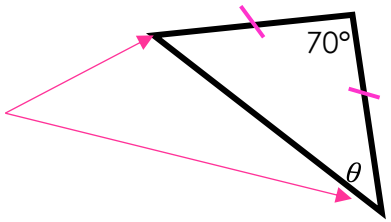
Polygon	Number of sides	Interior angle sum
Triangle	3	180°



To find the missing angle you can add the two angles that are there and subtract the answer from 180°

$180° - (23° + 70°) = 87°$

$x = 87°$



These two angles are the same.

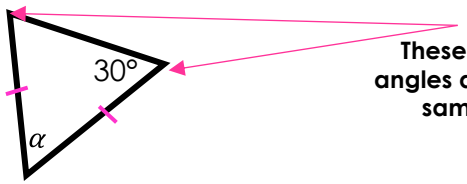
Here you need to do:

$180° - 70° - 110°$

Then the two remaining angles are the same.

$110° \div 2 = 55°$

$\theta = 55°$



These two angles are the same.

Here the base angles are the same, so you have $2 \times 30°$

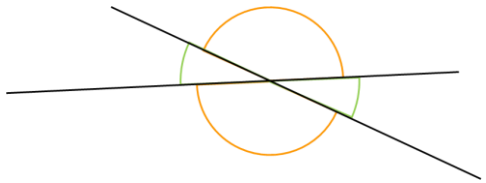
$2 \times 30° = 60°$

$180° - 60° = 120°$

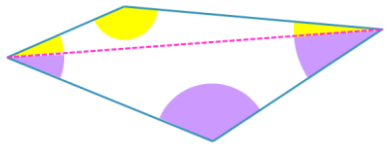
$\alpha = 120°$

Keyword/Skill	Definition/Tips
Angle	The amount of turn between two rays called arms meeting at a common point called vertex.
Vertically opposite	Pair of angles directly opposite to each other, formed by intersection of straight lines.
Reflex	Any angle that measures more than 180 degrees but less than 360 degrees.
Parallel	Equidistant lines, that is, exactly the same distance apart and never touching.
Perpendicular	Meeting or crossing at a right angle.
Vertices	A vertex (plural is vertices) is made when two lines meet.
Polygon	Two-dimensional closed shapes with straight sides are called polygons.
Acute	Any angle that measures less than 90 degrees.
Obtuse	Any angle that measures between 90 degrees and 180 degrees.
Right angle	Any angle that measures exactly 90 degrees.

Vertically opposite angles are equal



Year 10 – Maths – Unit GM2 – Angles & Polygons

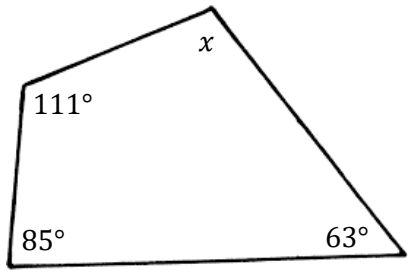


Shapes with four sides are called quadrilaterals.

Here is a quadrilateral.

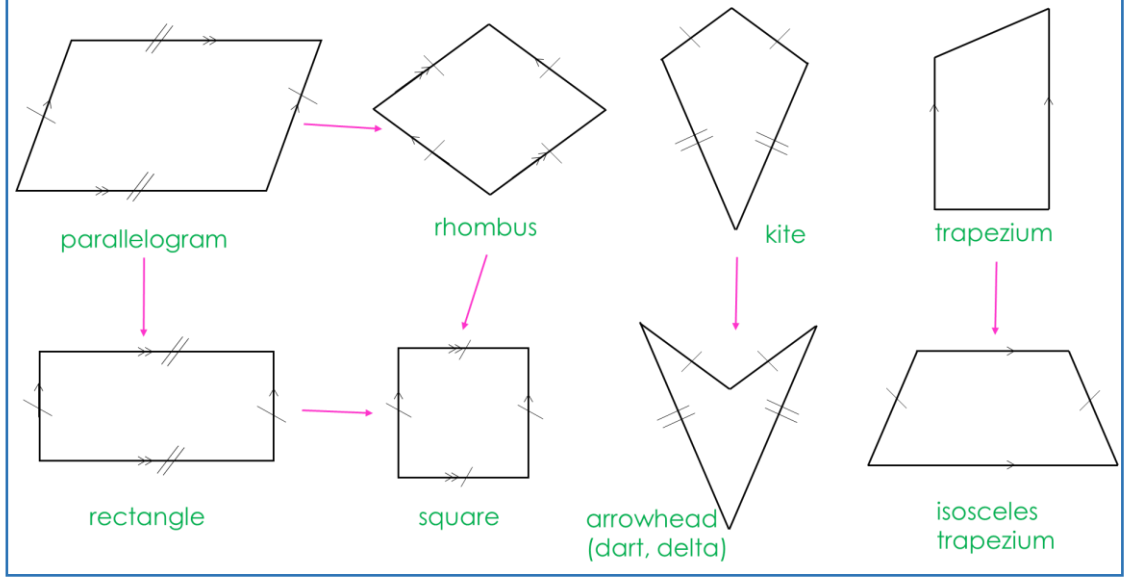
It can be split into two triangles.

This means its interior angles sum to $180° + 180° = 360°$



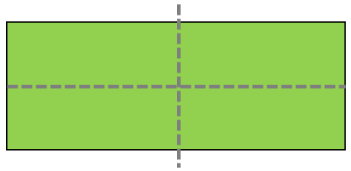
To find the missing angle you can add the three angles that are there and subtract the answer from 360°

$$360° - (63° + 111° + 85°) = 87°$$
$$x = 101°$$



Reflective symmetry

Where one half of a shape reflects exactly onto the other half.



This rectangle has two reflective lines of symmetry

Rotational symmetry

Where a shape rotates exactly onto itself more than once in a full turn.



This rectangle has rotational symmetry order 2.

A shape that only fits onto itself after one full turn has no rotational symmetry.

This means a shape can have no rotational symmetry or rotational symmetry of order 2, 3, 4, 5, …

Keyword/Skill	Definition/Tips
Angle	The amount of turn between two rays called arms meeting at a common point called vertex.
Vertically opposite	Pair of angles directly opposite to each other, formed by intersection of straight lines.
Reflex	Any angle that measures more than 180 degrees but less than 360 degrees.
Parallel	Equidistant lines, that is, exactly the same distance apart and never touching.
Perpendicular	Meeting or crossing at a right angle.
Vertices	A vertex (plural is vertices) is made when two lines meet.
Polygon	Two-dimensional closed shapes with straight sides are called polygons.
Acute	Any angle that measures less than 90 degrees.
Obtuse	Any angle that measures between 90 degrees and 180 degrees.
Right angle	Any angle that measures exactly 90 degrees.

Year 10 – Maths – Unit GM2 – Angles & Polygons

Fingertip facts: what I need to learn by heart

Polygon	Number of sides	Interior angle sum
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°
Heptagon	7	900°
Octagon	8	1080°
Nonagon	9	1260°
Decagon	10	1440°

Notice that the interior angle sum increases by 180° each time.

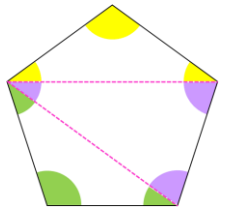
We know that the interior angles in a triangle sum to 180°.

We know that the interior angles in a quadrilateral sum to 360°.

Here is a pentagon.
We can split it into 3 triangles.

+ 180°

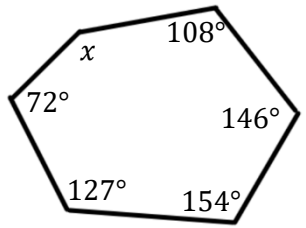
+ 180°



The interior angles in the pentagon sum to
180° + 180° + 180° = 540°.

For every side you add, add on another 180° to the interior angle sum.

Example:
Here is an irregular hexagon.

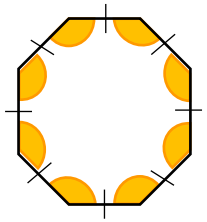


The sum of the interior angles of a hexagon is 720°.

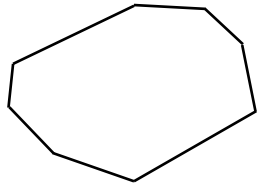
$720° - (108° + 146° + 154° + 127° + 72°)$

$x = 113°$

When all the sides are the same length and all the angles the same size, we say a polygon is regular.



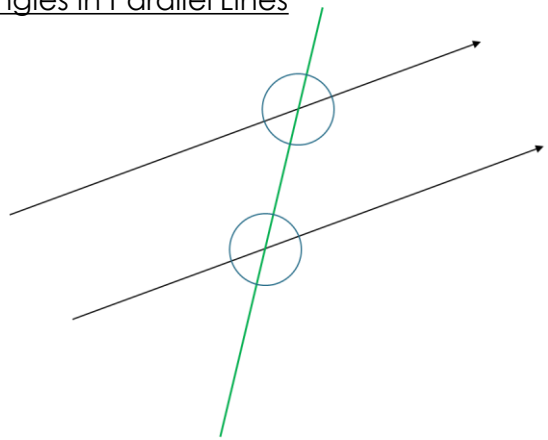
If any of the sides or angles are different, it is irregular.



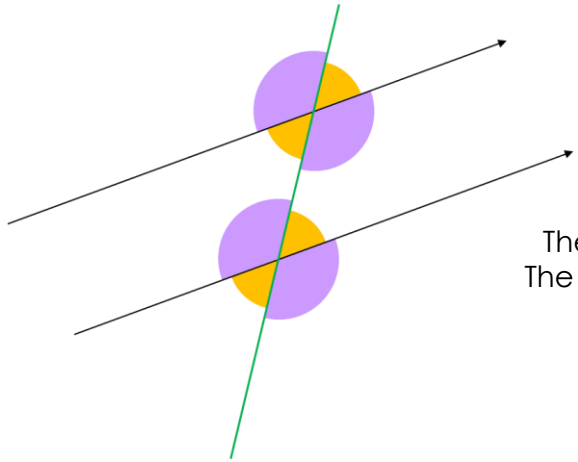
Keyword/Skill	Definition/Tips
Angle	The amount of turn between two rays called arms meeting at a common point called vertex.
Vertically opposite	Pair of angles directly opposite to each other, formed by intersection of straight lines.
Reflex	Any angle that measures more than 180 degrees but less than 360 degrees.
Parallel	Equidistant lines, that is, exactly the same distance apart and never touching.
Perpendicular	Meeting or crossing at a right angle.
Vertices	A vertex (plural is vertices) is made when two lines meet.
Polygon	Two-dimensional closed shapes with straight sides are called polygons.
Acute	Any angle that measures less than 90 degrees.
Obtuse	Any angle that measures between 90 degrees and 180 degrees.
Right angle	Any angle that measures exactly 90 degrees.

Year 10 – Maths – Unit GM2 – Angles & Polygons

Angles in Parallel Lines

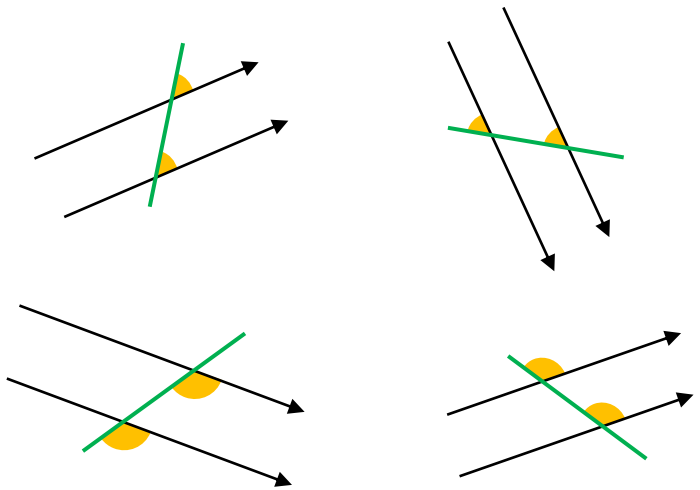


When we draw a transversal (line) through a set of parallel lines, two vertices are made.



The two vertices contain identical angles. The obtuse angles are equal, and the acute angles are equal.

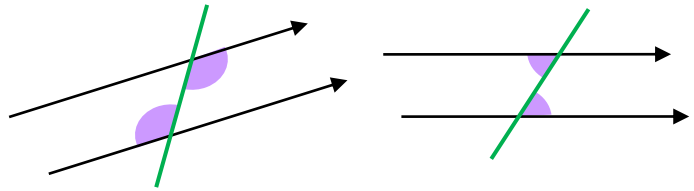
Corresponding angles



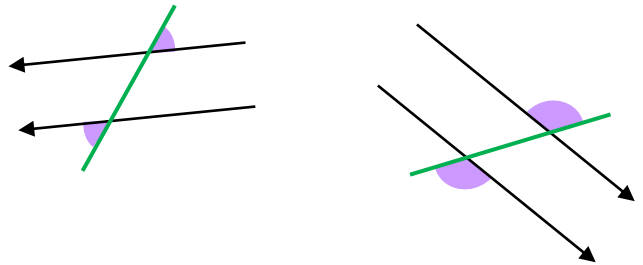
They are on the same side of the transversal and are equal.

Alternate angles

Alternate interior



Alternate exterior



They are on alternate sides of the transversal and are equal.

Keyword/Skill	**Definition/Tips**
Angle	The amount of turn between two rays called arms meeting at a common point called vertex.
Vertically opposite	Pair of angles directly opposite to each other, formed by intersection of straight lines.
Reflex	Any angle that measures more than 180 degrees but less than 360 degrees.
Parallel	Equidistant lines, that is, exactly the same distance apart and never touching.
Perpendicular	Meeting or crossing at a right angle.
Vertices	A vertex (plural is vertices) is made when two lines meet.
Polygon	Two-dimensional closed shapes with straight sides are called polygons.
Acute	Any angle that measures less than 90 degrees.
Obtuse	Any angle that measures between 90 degrees and 180 degrees.
Right angle	Any angle that measures exactly 90 degrees.

Year 10 – Maths – Unit GM2 – Angles & Polygons

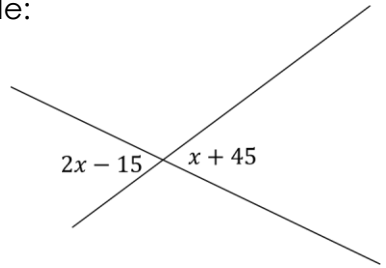
Solving Equations Using Angle Facts

You can use all the facts in the unit to form and solve equations too.

You first need to think, 'what angle facts can I apply here?'

Once you have done that, you can set up an equation.

Example:



The fact we need here is vertically opposite angles are equal.

Therefore, we can form an equation:

$$2x - 15 = x + 45$$

$$x - 15 = 45$$

$$x = 60$$

Bearings

Bearings are a form of directions using angles.

They follow three rules:

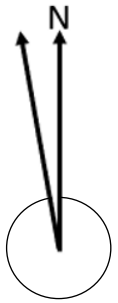
- Always measure from the North Line.



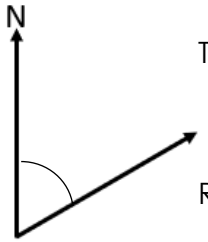
- Always measure clockwise.



- Bearings are always three figures.

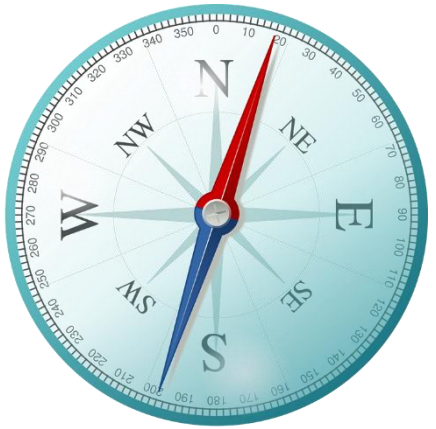


The bearing shown here would be $350°$



The bearing shown here would be $060°$

Remember it needs to be three digits.



Keyword/Skill	**Definition/Tips**
Angle	The amount of turn between two rays called arms meeting at a common point called vertex.
Vertically opposite	Pair of angles directly opposite to each other, formed by intersection of straight lines.
Reflex	Any angle that measures more than 180 degrees but less than 360 degrees.
Parallel	Equidistant lines, that is, exactly the same distance apart and never touching.
Perpendicular	Meeting or crossing at a right angle.
Vertices	A vertex (plural is vertices) is made when two lines meet.
Polygon	Two-dimensional closed shapes with straight sides are called polygons.
Acute	Any angle that measures less than 90 degrees.
Obtuse	Any angle that measures between 90 degrees and 180 degrees.
Right angle	Any angle that measures exactly 90 degrees.

Year 10 – Maths – Unit GM3 – Area

My mathematical journey

What do I need to remember from before?

- Area of a rectangle (KS2)
- Area of rectilinear shapes (NP3)
- Perimeter (KS2 & NP2)
- Types of polygons (GM2)

What will I learn about in this unit?

- Units of length and of area
- Area of a triangle
- Area of quadrilaterals
- Area of a circle

Where does this lead?

- Area and circumference of a circle (GM6)
- Surface area and volume (GM8)
- Advanced area (GM9)
- Solving geometric problems (GM11)

Units of Measurement

Units for perimeter:

mm, cm, m, km, etc.

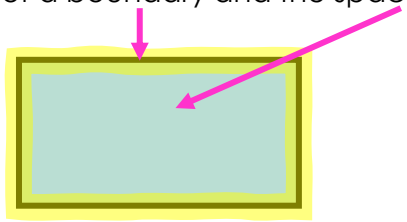
Units for area:

mm^2, cm^2, m^2, km^2, etc.

Area & Perimeter

The perimeter of a 2D shape is a measure of the length of its boundary.

Any 2D shape is made up of a boundary and the space inside.



The space inside the shape is its area.

Area of a Rectangle

Area of a rectangle = base × perpendicular height.

h

$$A = bh$$

b

$4cm$ | $40cm^2$ | $10cm$

The base is $10cm$

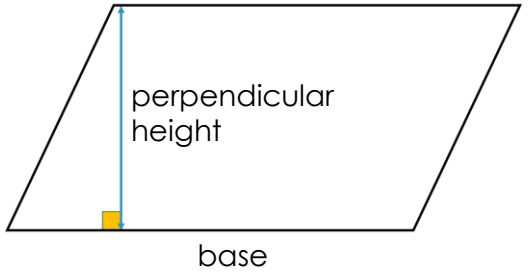
The height is $4cm$

$4cm \times 10cm = 40cm^2$

Keyword/Skill	**Definition/Tips**
Perimeter	Distance around the edge of a shape. Start and finish at the same point.
Area	The amount of space inside a shape. Measured in square units, such as cm^2, mm^2, m^2 …
Compound	Where more than one shape have been stuck together to form a new shape.
Perpendicular height	The height of a shape which is at a right angle to its base.
Radius	The length from the centre of a circle to its edge.
Diameter	The length straight across the centre of a circle from edge to edge. Double the radius.

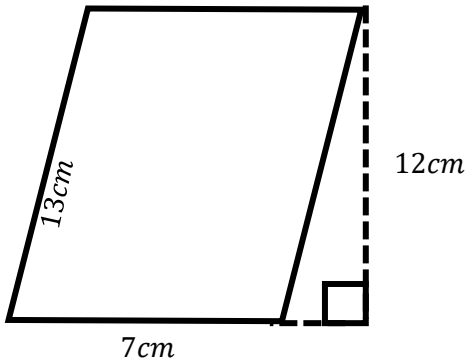
Year 10 – Maths – Unit GM3 – Area

Area of a Parallelogram



The area of a parallelogram is:
base × perpendicular height

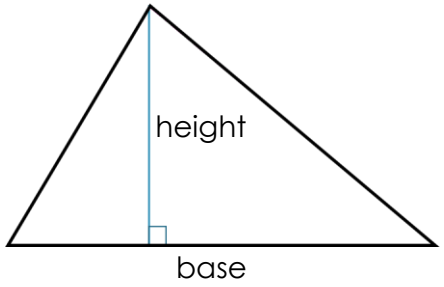
$$A = bh$$



Make sure you use the perpendicular height.
In this example, the base is $7cm$ and the perpendicular height is $12cm$

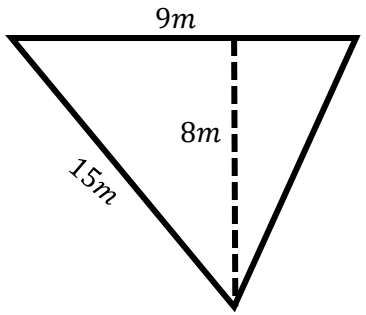
$$A = 7cm \times 12cm = 84cm^2$$

Area of a Triangle



The area of a triangle is:
$\frac{1}{2}$ × base × perpendicular height

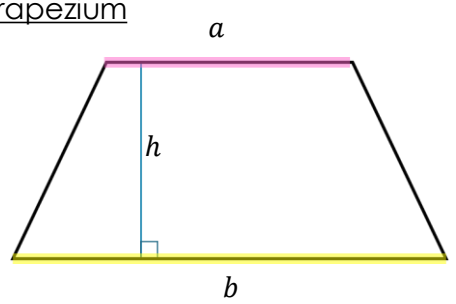
$$A = \frac{1}{2}bh$$



Make sure you use the perpendicular height.
In this example, the base is $9m$ and the perpendicular height is $8m$

$$A = \frac{1}{2} \times 9m \times 8cm = 36m^2$$

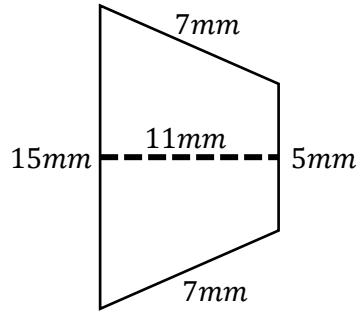
Area of a Trapezium



a and b are the set of parallel lines.
h is the perpendicular height.

The area of a trapezium is:
$\frac{1}{2}$ × (sum of the parallel lines) × perpendicular height

$$A = \frac{1}{2}(a + b)h$$



Make sure you use the set of parallel lines and the perpendicular height.
In this example, the base is $9m$ and the perpendicular height is $8m$

$$A = \frac{1}{2}(15mm + 5mm) \times 11mm = 110mm^2$$

Year 10 – Maths – Unit GM3 – Area

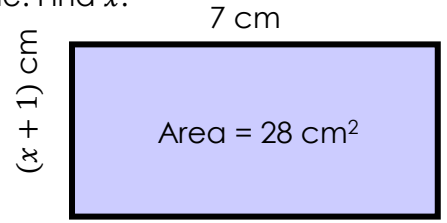
Solving Geometric Problems

We can use perimeter and area with algebra to solve more complex problems.

If you see area or perimeter questions with algebraic expressions,

1. Ask yourself, *what fact can I apply here?*
2. Set up and solve an equation using that fact

Example: Find x.



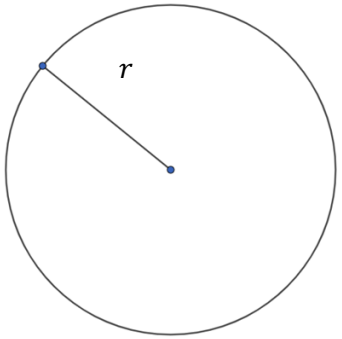
We know the area is base × height.

$$7 \times (x + 1) = 7x + 7$$

This is an expression for the area. Since we know what the area is, we can equate these two things.

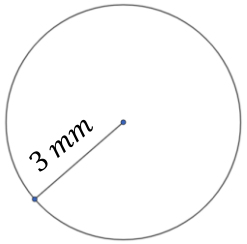
$$7x + 7 = 28$$
$$-7 \qquad -7$$
$$7x = 21$$
$$\div 7 \qquad \div 7$$
$$x = 3$$

Area of a Circle



The area of a circle is:
pi × radius²

$$A = \pi r^2$$

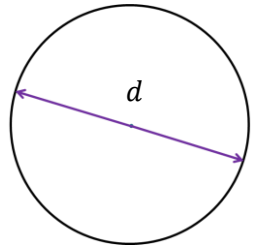


Make sure you use the radius (you may be given the diameter and need to half it to get the radius).
In this example, the base is $3mm$.

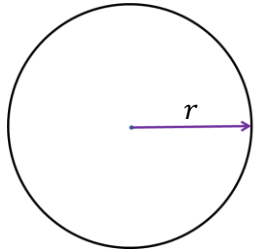
$$A = \pi \times 3^2 = 9\pi\ mm^2 = 28.27mm^2\ (2d.p.)$$

This is in terms of pi (π).

This is rounded to 2 decimal places.



A line that through the centre of a circle from the edge to the other edge is a diameter.
A diameter is double the radius.



A line that goes from the centre of a circle to its edge is called its radius.
A radius is half of the diameter.

Keyword/Skill	**Definition/Tips**
Perimeter	Distance around the edge of a shape. Start and finish at the same point.
Area	The amount of space inside a shape. Measured in square units, such as cm^2, mm^2, m^2 …
Compound	Where more than one shape have been stuck together to form a new shape.
Perpendicular height	The height of a shape which is at a right angle to its base.
Radius	The length from the centre of a circle to its edge.
Diameter	The length straight across the centre of a circle from edge to edge. Double the radius.

Year 10 – Maths – Unit NP1 – Place Value & Number Line

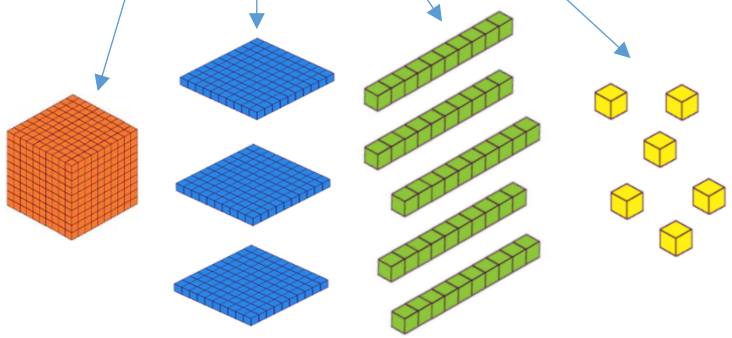
Place value

This is the number system we use every day.

Base 10 **place value** has a relationship of × 10 between columns, moving from right to left.

We can use a place value chart to help us read and write numbers in base 10.

Th Thousands	H Hundreds	T Tens	U Units	•	$\frac{1}{10}$ Tenths	$\frac{1}{100}$ Hundredths
1	3	5	6			



100,000	10,000	1,000	100	10	1	•	0.1	0.01	0.001	0.000 1	0.000 01
Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	•	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths
3	5	4	9	4	3	•					

This number is written as
three hundred and fifty-four thousand, nine hundred and forty-three.

Reading and Writing Numbers

0 – Zero
1 – One
2 – Two
3 – Three
4 – Four
5 – Five
6 – Six
7 – Seven
8 – Eight
9 – Nine

10 – Ten
11 – Eleven
12 – Twelve
13 – Thirteen
14 – Fourteen
15 – Fifteen
16 – Sixteen
17 – Seventeen
18 – Eighteen
19 – Nineteen

20 – Twenty
30 – Thirty
40 – Fourty
50 – Fifty
60 – Sixty
70 – Seventy
80 – Eighty
90 – Ninety

Midpoints and medians



The midpoint of two numbers is exactly halfway between them. To find it quickly, we add together the endpoints and the halve the answer.

When we are given a list of numbers, in order, the middle number in the list is called the median.
If our list contains an even number of numbers, then there will be two numbers in the middle. The median is the midpoint of these two (which isn't actually in the list!)

Positive and negative integers

We can still draw vectors to represent negative numbers.



This is the number 4. Its sign is positive.
This is the number -4 ("negative 4"). Its sign is negative.
What is the same about them? What is different about them?

My mathematical journey

What do I need to remember from before?
- Place value of numbers up to 10 000 000 (KS2)
- Rounding numbers to the nearest 10, 100, 1000, 10 000 and 100 000 (KS2)
- Rounding decimals to 1, 2 or 3 decimal places (KS2)
- Ordering negative numbers on a number line (KS2)
- Multiplying and dividing numbers by 10, 100 and 1000

What will I learn about in this unit?
- Writing integers and decimals in expanded form and words
- Ordering numbers
- Rounding to decimal places and to significant figures
- Converting metric units
- Finding the midpoint of two numbers
- Finding the median of discrete data

Where does this lead?
- Addition & subtraction (NP2)
- Multiplication & division (NP3)
- Percentages, fractions & decimals (NP8)
- Estimation (NP9)
- Analysing discrete data (SP1)
- Using units of measure (all GM units and many SP units)
- Standard form (NP12)
- Indices & surds (NP15)

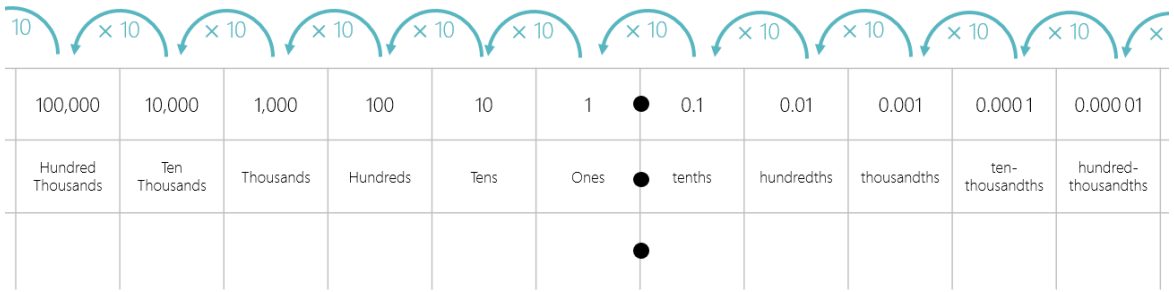
Key words & symbols

Word	Explanation
number	a value or a quantity used to count or measure
digit	a symbol we use to make numbers, such as "0" or "9"
numeral	a number written with digits, such as "213" or "0.5"
integer	a "whole" number (with no decimal part), such as 15 or 510, but not 2.5
base 10	our numeral system, where each column is worth a different power of 10
decimal	means "base 10" but more often used for non-integers written like this: 2.5 or 38.7
less than	numbers further left on the number line
greater than	numbers further right on the number line
ascending	going up
descending	going down

Symbol	How to read it
<	is less than
>	is greater than
≤	is less than or equal to
≥	is greater than or equal to
=	is equal to
≠	is not equal to
≈	is approximately equal to

Year 10 – Maths – Unit NP1 – Place Value & Number Line

Multiplying and dividing by 10, 100, 1000, etc



Every column is ten times greater than the one before it.

This means when we × 10, all digits move one place to the left.

If we × 100, they move two places left, × 1000 moves three places left, etc.

Common metric units

Some common ones. (modern)

Quantity to measure	Metric units
length	millimetre (mm) centimetre (cm) metre (m) kilometre (km)
mass	gram (g) kilogram (kg)
capacity	millilitre (ml) litre (l)

mm	cm	m
1000	100	1

m	km
1000	1

s	min	hrs
3600	60	1

mg	g	Kg
1000	100	1

ml	c l	l
1000	100	1

hrs	day
24	1

The metric units are part of the **système international (SI)** of units.

Convert:

5 m to cm

5m = 500cm

3500 g to kg

3500 g = 3.5 kg

900 ml to l

900 ml = 0.9l

20 km to m

20 km = 20 000 m

Rounding: nearest 10, 100, 1000, etc

What is 36 458 to the nearest 1000?

36 458 → 37 000 / 36 000

36 458 ≈ 36 000

What is 36 458 to the nearest 100?

36 458 → 36 500 / 36 400

36 458 ≈ 36 500

Rounding: to decimal places

What is 0.77 to one decimal place?

0.77 → 0.8 / 0.7

0.77 ≈ 0.8

What is 0.638 to two decimal places?

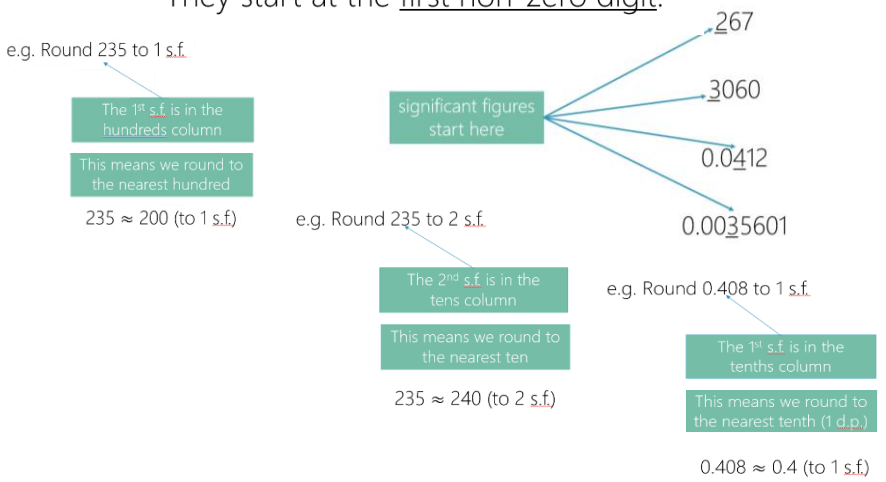
0.638 → 0.64 / 0.63

0.638 ≈ 0.64

Rounding: to significant figures

Significant figures are digits in a number.

They start at the first non-zero digit.



Year 10 – Maths – Unit NP2 – Addition & Subtraction

$5 + 8 = 13$

summand + summand = sum

summand, summand, sum

$8 - 5 = 3$

minuend – subtrahend = difference

minuend, subtrahend, difference

Additive Inverses

The numbers in a zero pair are called additive inverses of each other.

$5 + (-5) = 0$

Additive Inverse of 5 is -5

Column method for addition and subtraction

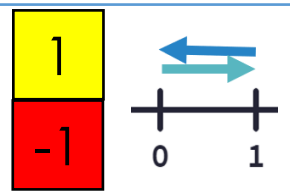
$$\begin{array}{r} 429 \\ \underline{283}\ - \end{array}$$

Make sure you line up digits using their place values.

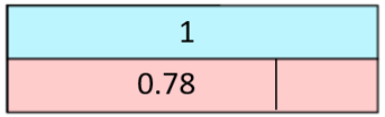
$$\begin{array}{r} 4.29 \\ \underline{0.283}\ + \end{array}$$

Zero Pairs

1 and -1 are called a zero pair because their sum is 0.



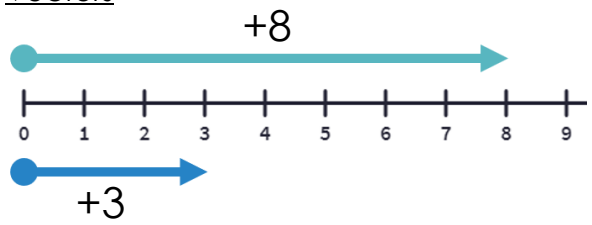
Number Bonds



Knowledge of number bonds helps with quick addition and subtraction.

Make all the digits after the decimal point up to 9, but make the last up to 10.

Vectors



Vectors represent the magnitude (size) of a number on a number line.

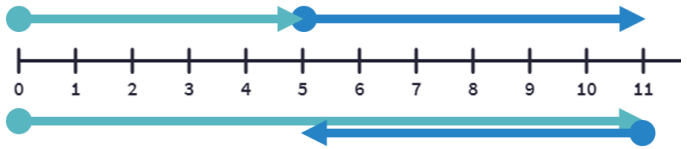
When we add or subtract numbers along a number line, we place then end to end.

Inverse Operations

Inverse operations are the opposite of one another, so they undo one another.

$5 + 6 = 11$

$11 - 6 = 5$



My mathematical journey

What do I need to remember from before?

- Place value (NP1)
- Vectors on a number line (NP1)
- Adding and subtracting whole numbers with pen and paper and mentally (KS2)
- Using rounding to check answers to calculations (KS2)

What will I learn about in this unit?

- Addition and subtraction with integers and decimals
- Commutativity & mental methods with integers and decimals
- Number bonds, complements, working with decimals
- Vectors, inverse operations, equality and zero pairs
- Perimeter
- Angle facts
- Mean and range

Where does this lead?

- Order of operations (NP5)
- Directed numbers (NP6)
- Simplifying expressions (A1)
- Adding & subtracting fractions (NP7)
- Solving linear equations (A2)
- Adding & subtracting numbers in standard form (NP12)
- Adding and subtracting surds (NP15)

Keyword/Skill	Definition/Tips
Integer	Whole number including 0 and negative numbers. No fractions or decimals.
Place Value	Hundreds, Tens, Ones, Tenths, Hundredths…
Negative numbers	Number less than zero. Can be integer, decimal or fraction, e.g. -2, -4.7, $-\frac{1}{2}$
Positive numbers	Numbers bigger than zero. Can be integer, decimal or fraction, e.g. 5, 3.6, $\frac{2}{5}$
Summand	The summands are the numbers being added
Sum	The answer when summands are added together
Zero pair	Two values with equal magnitude (size) but opposite signs, so they give a sum of 0, e.g. 2 + - 2 = 0
Complement	The complement of a decimal is the number you add to get to 1.
Difference	Answer after subtraction of two values
Column Addition	Standard written method of addition
Minuend	When we subtract, the starting/first number is the minuend
Subtrahend	When we subtract, the amount to be subtracted is the subtrahend

Year 10 – Maths – Unit NP2 – Addition & Subtraction

Commutativity

An operation is commutative if it can be applied to two numbers in **any order**

Examples

3 x 4 = 12 is the same as 4 x 3 = 12

2 + 5 = 7 is the same as 5 + 2 = 7

Addition and multiplication are commutative

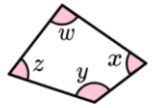
Non - Examples

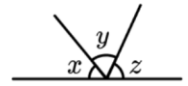
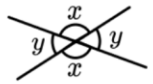
15 ÷ 5 is not the same as 5 ÷ 15

10 - 3 is not the same as 3 – 10

Division and subtraction are not commutative

Basic Angle Facts

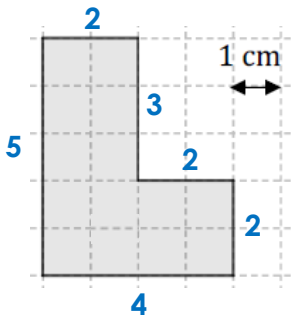
Angle Rule	Description	Diagram
Angles in a triangle	The sum of angles in a triangle is 180°. $x + y + z = 180$	
Angles in a quadrilateral	The sum of angles in a quadrilateral is 360°. $w + x + y + z = 360$	

Angle Rule	Description	Diagram
Angles on a straight line	The sum of angles on a straight line is 180°. $x + y + z = 180$	
Angles at a point	The sum of angles at a point is 360°. $w + x + y + z = 360$	
Vertically opposite angles	Vertically opposite angles are equal in size.	

Perimeter

Perimeter is the **distance** all the way around the **edge** of a shape.

By **counting edges** of **squares** along the **sides** of a shape on a grid, we can work out the perimeter of that shape.



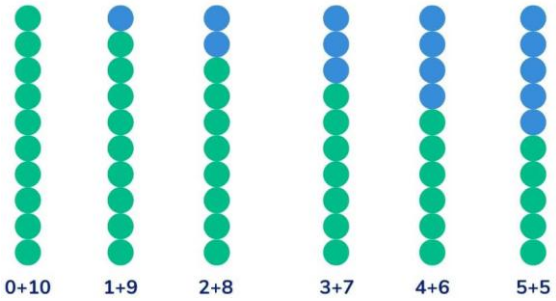
The perimeter of this L shape is
2 + 3 + 2 + 2 + 4 + 5 = 18 cm

Number Bonds

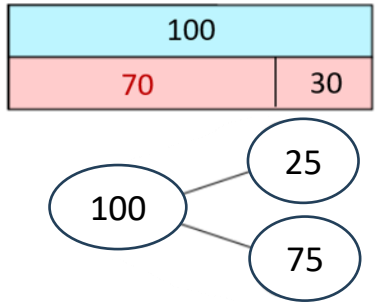
Number bonds are pairs of numbers that can be added together to make another number, like 10, 100, 1000 etc.

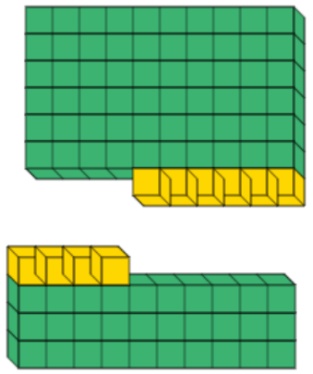
We can also have pairs of decimal numbers that add to make 1. This is called the **complement of a decimal.**

Number Bonds to 10

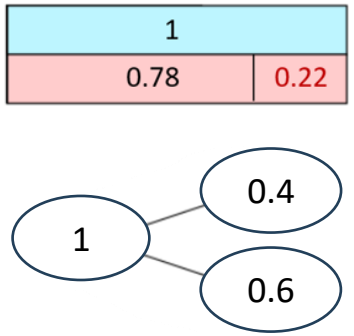


Number Bonds to 100





Complement of a Decimal/Number Bonds to 1



Keyword/Skill	Definition/Tips
Commutative	An operation that, in any order, gives the same result, e.g. 4 x 2 = 8 and 2 x 4 = 8, 5 + 2 = 7 and 2 + 5 = 7
Perimeter	The distance around a shape.
Number bonds	Pairs of numbers that add to make another given number.
Angle	The measure of a turn
Complement of a decimal	The decimal number you would add to another decimal number to make 1.

Year 10 – Maths – Unit NP3 – Multiplying & Dividing

My mathematical journey

What do I need to remember from before?	What will I learn about in this unit?	Where does this lead?
Place value (NP1)	Multiplication and division with integers and decimals	Powers, roots and primes (NP4)
Vectors on a number line (NP1)	Area models for multiplication	Order of operations (NP5)
Multiplying and dividing on paper and mentally (KS2)	Multiples and factors	Directed numbers (NP6)
Using rounding to check answers to calculations (KS2)	Multiplying to stretch	Fractions (NP7)
	Area and volume	Percentages (NP8)
		Proportional reasoning (NP10)

Commutativity

An operation is commutative if it can be applied to two numbers in **any order**

Example

$3 \times 4 = 12$ is the same as $4 \times 3 = 12$

$2 + 5 = 7$ is the same as $5 + 2 = 7$

Addition and multiplication are commutative

Non - Example

$15 \div 5$ is not the same as $5 \div 15$

$10 - 3$ is not the same as $3 - 10$

Division and subtraction are not commutative

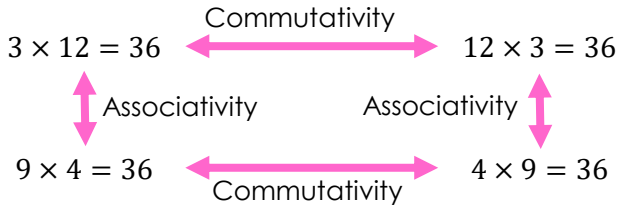
Associativity

Multiplication is associative. This means you can 'split up' parts of it and join the numbers in different ways. We call it decomposing and recomposing.

$3 \times 12 = 36$

$3 \times 3 \times 4 = 36$

$9 \times 4 = 36$



This shows us that 3×12 and 9×4 are the same calculation.

Keyword/Skill	Definition/Tips
Integer	Whole number including 0 and negative numbers. No fractions or decimals.
Associativity	Getting the same result regardless of the grouping. E.g. 2 x 3 x 4 = 24 4 x 3 x 2 = 24
Commutative	An operation that, in any order, gives the same result, e.g. 4x2=8 and 2x4=8, 5+2=7 and 2+5=7
Product	Multiply

Multiplying – Grid Method

When multiplying two integers, a grid method is way to multiply which helps limit our mistakes.

Example:

35×748

Make sure you align your place value columns correctly!

×	700	40	8
30	21000	1200	240
5	3500	200	40

21000
3500
1200
240
200
40
26180

$$35 \times 748 = 26180$$

Multiplying Decimals

When we multiply decimals, we can first make an easier, related calculation without decimals, then reverse our changes to get the final answer.

Example:

$$0.5 \times 0.6$$

$\times 10$ $\times 10$

$$5 \times 6 = 30$$

$\div 100$

$$0.5 \times 0.6 = 0.3$$

As we have multiplied by 10 twice, we need to divide by 100 (same as dividing by 10 twice).

You will sometimes need to use grid method

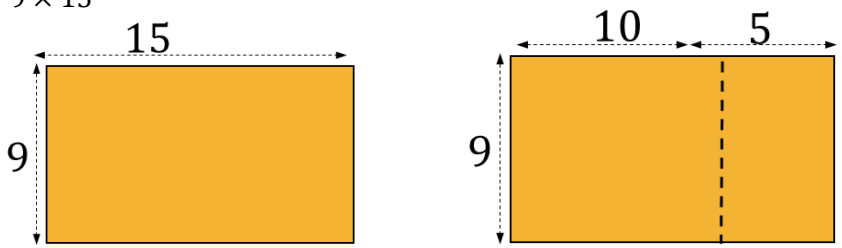
Year 10 – Maths – Unit NP3 – Multiplying & Dividing

Distributive Property

A way of splitting up a calculation to make it more manageable

Example:

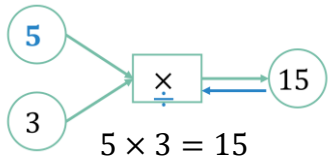
9×15



$$9 \times (10 + 5) = 9 \times 10 + 9 \times 5$$
$$= 90 + 45$$
$$= 135$$

Fact Families

From this diagram, we can see a family of facts.



$5 \times 3 = 15$

$3 \times 5 = 15$

$15 \div 3 = 5$

$15 \div 5 = 3$

Multiples & Lowest Common Multiple

Multiples can be thought of as numbers in a times table.

e.g. The multiples of 5 are the numbers in the 5 times table.

Multiples of 5: 5, 10, 15, 20, 25, 30, 35, …

Numbers have common multiples; these are multiples they share.

Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, …

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, …

The common multiples are: 20, 40, 60, …

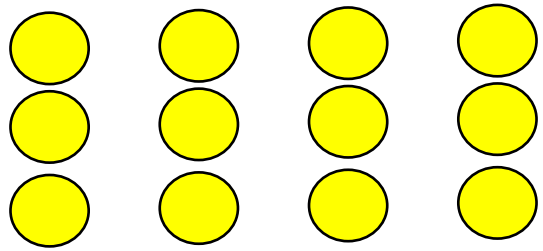
Questions usually ask what the lowest common multiple is.

Example:
What is the lowest common multiple of 12 and 16.
Multiples of 12: 12, 24, 36, 48, 60, 72, 84, 96, …
Multiples of 16: 16, 32, 48, 64, 80, 96, 112, 128, …
The lowest common multiple is 48. Make sure you check the list carefully.
96 is a common multiple but not the **lowest** common multiple.

Repeated Addition & Scaling

Multiplication can be represented in two ways, repeated addition & scaling

Repeated Addition



$3 + 3 + 3 + 3$

$= 4 \times 3 = 12$

There are 4 lots of 3

Scaling



We want 4 lots of 3



$= 4 \times 3 = 12$

There are 4 lots of 3

Year 10 – Maths – Unit NP3 – Multiplying & Dividing

Division

We are going to focus on using short division. The best method to use is bus stop.

Example:

$$471 \div 3 = \frac{471}{3}$$

$$3 \overline{)\,4\;7\;1}$$

You don't need to write separate bus stops. It would all go in the same one like this.

How many 3's go into 4?

$$3 \overline{)\,4^{1}7\;1} \quad 1$$

4 remainder 1

How many 3's go into 17?

$$3 \overline{)\,4^{1}7^{2}1} \quad 1\;5$$

5 remainder 2

How many 3's go into 17?

$$3 \overline{)\,4^{1}7^{2}1} \quad 1\;5\;7$$

7

$$471 \div 3 = 157$$

Dividing Decimals

When the divisor is integer, we can use our knowledge of multiplication or short division.

$$8.36 \div 4 =$$

$$4 \overline{)\,8.3^{3}6} \quad 2.0\;9$$

Make sure you keep the decimal in the correct place!

When the divisor is not an integer, we first make an easier calculation that has the same answer.

$$12 \div 0.3 = \frac{12}{0.3}$$

$$= \frac{12 \times 10}{0.3 \times 10}$$

$$= \frac{120}{3}$$

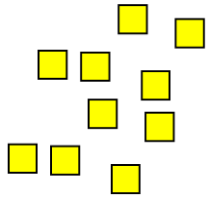
$$= 40$$

As we haven't changed the value of the fraction the answers are equivalent!

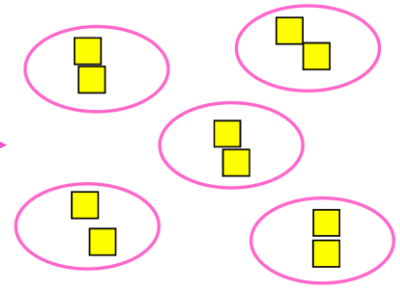
Representations of Division

There are three ways we can think of representing division. These are sharing, grouping & repeated subtraction. We will see the representations for $10 \div 5$

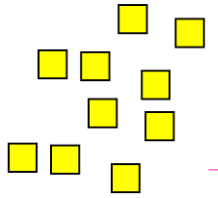
Sharing - $10 \div 5$



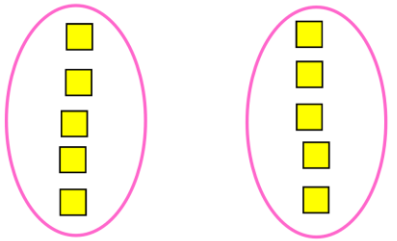
Think of this as 10 shared into 5 parts.



Grouping - $10 \div 5$



Think of this as the number 10 split into groups of size 5. The answer is the number of groups split into 5.



Scaling

We can instead think of this as the number of times we must subtract 5 from 10 to get to 0. The answer (2) is the number of times we subtract 5 to get to 0.



Year 10 – Maths – Unit NP3 – Multiplying & Dividing

Factors & Highest Common Factor

Factors can be thought of as the integers which multiply to make another.
e.g. 7 and 11 are factors of 77, because 7 × 11 = 77.
When writing the list of factors, make sure to be logical and list pairs until you have them all.
Example:
List all the factors of 24:
1, 24
2, 12
3, 8
4, 6
We can write them as an ordered list to check we have them all:
1, 2, 3, 4, 6, 8, 12, 24

Any numbers that are factors of two or more numbers are said to be **common factors** of those numbers.

Factors of 12:

1 x 12
2 x 6
3 x 4

} All the ways of making a product of 12.

1, 2, 3, 4, 6, and 12 are all the factors of 12.

Factors of 20:

1 x 20
2 x 10
4 x 5

} All the ways of making a product of 20.

1, 2, 4, 5, and 20 are all the factors of 12.

Both lists of factors here have 1, 2 and 4 included. Therefore, 1, 2 and 4 are common factors of 12 and 20.
You are usually asked what the highest common factor is. In this case the highest common factor of 12 and 20 is 4.
Make sure you check your list to get the highest common factor.

Factors & Highest Common Factor

Sometimes we solve problems by multiplying or dividing. We need to look for clues that tell us to multiply or divide.

If we are looking for "lots of" something, that often means *multiply*.
If we are looking to share something out, that often means *divide*.

In a train there are eight coaches each with 44 seats. How many seats are there on the train?

Here this would be a multiplication problem as there are 8 coaches with 44 seats which would be 8 'lots of' 44.

45 sweets are shared equally between 9 children. How many do they each get?

Here this would be a division problem as you are 'sharing' things out into groups.

Other Topics/Units this could appear in:

- Numbers, powers, rots, decimals and rounding
- Product of prime factors
- Multiples in context
- Factorising

Keyword/Skill	**Definition/Tips**
Integer	Whole number including 0 and negative numbers. No fractions or decimals.
Product	Multiply
Factor	Numbers we can multiply together to get another number.
Distributive Law	Multiplying a number by a group of numbers added together is the same as doing each multiplication.
Multiples	The result of multiplying a number by an integer (comes up in its timetable)
Common multiples	A number that is a multiple of two numbers
LCM	Smallest whole number that is a multiple of two numbers
Factors	An integer that divides the number exactly leaving no remainder
Factor pairs	A set of numbers that multiply to equal the number
HCF	The highest common factor (HCF) of two or more numbers is the largest number that is a factor of all of the given numbers.

Year 10 – Maths – Unit NP4 – Powers, Roots & Primes

Square Numbers

1, 4, 9, 16, 25, 36, 49, 64, …

These are called the square numbers because their area models are squares.
When we multiply a number by itself, we call this squaring the number.

Cube Numbers

1, 8, 27, 64, 125, 216, 343, …

These are called the cube numbers because their volume models are cubes.
When we multiply a number by itself, and then again, we call this cubing the number.

Using a Calculator

Some powers/roots will take too long to calculate so you will need to be confident using a calculator. You will need these buttons below.



Prime Numbers

A prime number is any positive integer that has exactly two different factors.
These factors are always itself and 1.

2 3 5 7 11 13 17
19 23 29 31 37 41
43 47 53 59 61 67
71 73 79 83 89 97

Index Notation

Index form is writing numbers in terms of their powers. It shows repeated multiplication.
Example:

$$2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

The inverse of an exponent is called a root. We show it with this symbol:

$$\sqrt{}$$

The inverse of squaring is called the square root, $\sqrt[2]{}$
The inverse of cubing is called the cube root, $\sqrt[3]{}$

Keyword/Skill	Definition/Tips
Product	Means multiply
Prime number	A number that has exactly two factors
Square number	A number multiplied by itself i.e. $3^2 = 9$
Cube number	The result of multiplying an integer by itself three times i.e. $2 \times 2 \times 2 = 2^3 = 8$
Index Form	Writing numbers in terms of powers E.g. $6 \times 6 \times 6 \times 6 = 6^4$ <- This is in index form
Prime Factor	A factor of a number that is also a prime number
Prime Factor Decomposition /Prime Factorisation	To write a number as a product of prime numbers Every number has a unique prime factorisation
Exponent	Another word for power. How many times you multiply a number by itself.
Root	The inverse of exponent.

My mathematical journey

What do I need to remember from before?

Multiplication and division (NP3)

Multiplying by composing and decomposing (NP3)

Multiples and factors (NP3)

What will I learn about in this unit?

Repeated multiplication

Powers and roots

Prime numbers

Composing and decomposing primes

Where does this lead?

Order of operations (NP5)

Directed numbers (NP6)

Quadratics (A11)

Index laws (NP15)

Exponential growth (NP16)

Year 10 – Maths – Unit NP4 – Powers, Roots & Primes

Index Laws

When we are multiplying numbers written in index form, and they have the same base, we can add the exponents. The reason we do this is shown here:

$$5^2 \times 5^4$$

We could write it out in full.

$$5^2 \times 5^4 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$$
$$= 5 \times 5 \times 5 \times 5 \times 5 \times 5$$
$$= 5^6$$

Example:

$$7^6 \times 7 \times 7^{10} = 7^{6+1+10} = 7^{17}$$

Since division is the inverse of multiplication, the law can be inverted too.
When we are dividing numbers written in index form, and they have the same base, we can subtract the exponents.

Example:

$$\frac{9^{10}}{9^4} = \frac{9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9}{9 \times 9 \times 9 \times 9}$$

$$= 9^6$$

Prime Factor Decomposition

Any number can be written as a **product of prime factors**. It is also called **Prime Factorisation** or **Prime Factor Decomposition.**

1) Start with the number at the top and split it into **factors** as shown.
2) Then do the same with the factors you have written.
3) If the number is a **prime number**, you can't break it down any further.
4) Keep going until you can't go any further (i.e. you are just left with prime numbers
5) Write these prime numbers out as a **product**.
6) If there is more than one of the same factor, you can write them as powers (**index form**).

No matter what numbers you choose for each step, you'll find the product of primes is exactly the same.

Example:
Express 420 as a product of its prime factors.

420 = 42 x 10

420 = 42 x 5 x 2

420 = 6 x 7 x 5 x 2

420 = 2 x 3 x 7 x 5 x 2

420 = 2 x 2 x 3 x 5 x 7

420 = 2^2 x 3 x 5 x 7

Keyword/Skill	Definition/Tips
Product	Means multiply
Prime number	A number that has exactly two factors
Square number	A number multiplied by itself i.e. 3^2 =9
Cube number	The result of multiplying an integer by itself three times i.e. 2 x 2 x 2 = 2^3 = 8
Index Form	Writing numbers in terms of powers E.g. 6 x 6 x 6 x 6 = 6^4 <- This is in index form
Prime Factor	A factor of a number that is also a prime number
Prime Factor Decomposition /Prime Factorisation	To write a number as a product of prime numbers Every number has a unique prime factorisation
Exponent	Another word for power. How many times you multiply a number by itself.
Root	The inverse of exponent.

Other Topics/Units this could appear in:

- Numbers, powers, roots, decimals and rounding
- Product of prime factors
- Multiples in context
- Index Laws
- Standard Index Form

Year 10 – Maths – Unit NP5 – Order of Operations

My mathematical journey

What do I need to remember from before?

Addition and subtraction (NP2)

Multiplication and division (NP3)

Exponents and roots (NP4)

What will I learn about in this unit?

Flexible calculating

The order of operations

Using visible and invisible brackets to break the order of operations

Where does this lead?

Directed numbers (NP6)

Substitution (A1, A2, A5)

Linear equations (A4)

Which Operations are Commutative?

				Commutative?
Multiplication	Multiplication	$2 \times 3 = 6$	$2 \times 3 = 3 \times 2$	YES
	Division	$\frac{6}{3} = 2$	$\frac{6}{3} \neq \frac{3}{6}$	NO
Addition	Addition	$2 + 3 = 5$	$2 + 3 = 3 + 2$	YES
	Subtraction	$5 - 3 = 2$	$5 - 3 \neq 3 - 5$	NO
Exponentiation	Exponents	$2^3 = 8$	$2^3 \neq 3^2$	NO
	Roots	$\sqrt[3]{8} = 2$	$\sqrt[3]{8} \neq \sqrt[8]{3}$	NO

The Operations

	Multiplication	Addition	Exponentiation
Operation	$2 \times 3 = 6$ $2 \cdot 3 = 6$	$2 + 3 = 5$	$2^3 = 8$
Inverse Operation	$6 \div 3 = 2$ $\frac{6}{3} = 2$	$5 - 3 = 2$	$\sqrt[3]{8} = 2$

Moving Numbers Around

As addition and multiplication are commutative. We can move numbers around the operations and get the same answer.
Subtraction and division are not commutative so when we move them around, we get a different answer.

Addition	Subtraction	Multiplication	Division
$12 + 3 + 5 = 12 + 5 + 3$	$12 - 3 - 5 = 12 - 5 - 3$	$3 \times 4 \times 5 = 3 \times 5 \times 4$	$24 \div 3 \div 2 = 24 \div 2 \div 3$
and	but	and	but
$12 + 3 + 5 = 5 + 3 + 12$	$12 - 3 - 5 \neq 5 - 3 - 12$	$3 \times 4 \times 5 = 5 \times 3 \times 4$	$24 \div 3 \div 2 \neq 2 \div 3 \div 24$
and	and	and	and
$12 + 3 + 5 = 3 + 12 + 5$	$12 - 3 - 5 \neq 3 - 12 - 5$	$3 \times 4 \times 5 = 4 \times 5 \times 3$	$24 \div 3 \div 2 \neq 3 \div 24 \div 2$

Keyword/Skill	**Definition/Tips**
Priority	The order of importance of a list of things. Higher priority means this must be done first.
Operation	A process in which a number, quantity, expression, etc., is altered according to set formal rules, such as those of addition, multiplication, and division.
Distributivity	A way of splitting up a calculation to make it more manageable.
Commutativity	An operation is commutative if it can be applied to two numbers in any order.
Exponentiation	Another word for power. This will include the inverse as well in roots.
Equivalent	Equal in value.
Variable	A symbol for a number we do not know yet.

Year 10 – Maths – Unit NP5 – Order of Operations

We now know the order in which we calculate.

This group is repeated multiplication, so we can think of it as "stronger" than multiplication.

This group is repeated addition, so we can think of it as "stronger" than addition.

- Exponentiation: Exponents, Roots
- Multiplication: Multiplication, Division
- Addition: Addition, Subtraction

Brackets break the order, so we must always look at them first.

Example:

$3 \times 5 \times 2 + 12 \div 3$

When we solve calculations, we look for the multiplication groups first (if there are no exponents or brackets)

$= 30 + 4$

$= 34$

Example:

All exponentiation should be worked out before other multiplication.

Exponentials first

$3 \times 2^3 + 5 \times 3$

Multiplication groups next

$3 \times 8 + 5 \times 3$

Addition groups last

$= 24 + 15$

$= 39$

Keyword/Skill	Definition/Tips
Priority	The order of importance of a list of things. Higher priority means this must be done first.
Operation	A process in which a number, quantity, expression, etc., is altered according to set formal rules, such as those of addition, multiplication, and division.
Distributivity	A way of splitting up a calculation to make it more manageable.
Commutativity	An operation is commutative if it can be applied to two numbers in any order.
Exponentiation	Another word for power. This will include the inverse as well in roots.
Equivalent	Equal in value.
Variable	A symbol for a number we do not know yet.

Example:

Brackets break the order. So, if we wanted to add before we multiply, we would put brackets around the addition calculation.

$3 \times 9 + 5$

No brackets here, so multiply first.

$27 + 5$

32

Different answers!

$3 \times (9 + 5)$

Do the calculation inside the brackets first

3×14

42

Year 10 – Maths – Unit NP6 – Directed Number

Comparing Positive and Negative Numbers



-4 is to the left of 4.
We say -4 is *less than* 4 and write $-4 < 4$.

4 is to the right of -4.
We say 4 is *greater than* -4 and write $4 > -4$.

Don't confuse *less than* with *smaller than*.
These two numbers have the same magnitude (size).

Don't confuse *greater than* with *bigger than*.
These two numbers have the same magnitude (size).

Number	What we write	What we say
6	6, +6, $^{+}6$	"six" "positive six"
−6	−6, $^{-}6$, (−6)	"negative six"

Lots of people say "minus six", but we will try to avoid this

Example

Write the numbers in order from least to greatest

3, −5, 1, 0, −2, 4

The number with the lowest value is the biggest negative number (−5).
The number with the greatest value is the biggest positive number (4).

Ordered list:

−5, −2, 0, 1, 3, 4

My mathematical journey

What do I need to remember from before?

- Addition and subtraction with integers and decimals (NP2)
- Multiplication and division with integers and decimals (NP3)
- Exponents and roots (NP4)
- Order of operations (NP5)

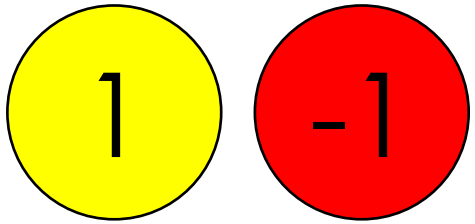
What will I learn about in this unit?

- Direction of numbers
- Using negative numbers
- Calculating with negative numbers

Where does this lead?

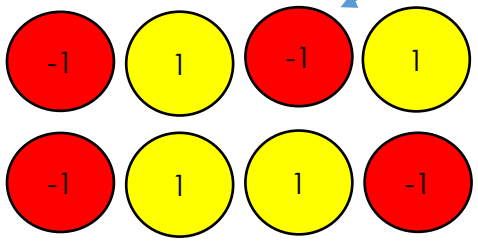
- Algebraic expressions (A2, A3)
- Linear equations (A4)
- Formulae (A5)
- Quadratic expressions (A11)

Look back at NP2 for more help with zero pairs.
You will see these double-sided counters a lot in this unit too:



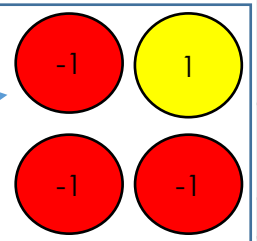
Keyword/Skill	Definition/Tips
Integer	Whole number including 0 and negative numbers. No fractions or decimals.
Negative numbers	Number less than zero. Can be integer, decimal or fraction, e.g. -2, -4.7, $-\frac{1}{2}$
Positive numbers	Numbers bigger than zero. Cam be integer, decimal or fraction, e.g. 5, 3.6, $\frac{2}{5}$
Multiple	A multiple of a number is all the numbers in that times table
Commutative	An operation that, in any order, gives the same result, e.g. 4x2=8 and 2x4=8, 5+2=7 and 2+5=7
Equal pairs	Two sums that have the same answer, e.g. -5+-2=-7 and -5-2=-7, 5- $^{-}$2=7 and 5+2=7
Solution	Answer to a problem
Sum	Total of a series of numbers
Product	Multiply
Difference	Answer after subtraction of two value

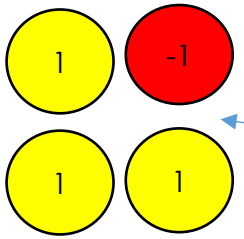
Zero Pairs and More



This set of numbers make a zero pair, as there are the same amount of positives as negatives.
The additive inverse of 4 is -4. That is what these counters show.

This number would be negative as there are more negative counters than positive counters. The overall value would be −2.





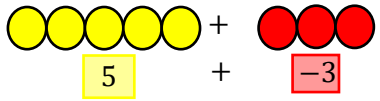
This number would be positive as there are more positive counters than negative counters. The overall value would be 2.

Year 10 – Maths – Unit NP6 – Directed Number

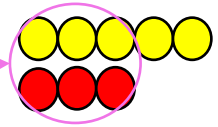
Adding Directed Number

We can use the tiles to help us add directed numbers

$5 + -3$



Adding is grouping them together. When we group them together we would make two zero pairs.

Zero Pairs 

This means we would be left with 2 

$5 + -3 = 2$

We use the tiles to help as a representation of the sum.

You need to grow to not use the tiles but still use the ideas they represent.

Example:

$8 + -10$

I know I would have 8 zero pairs as we are grouping together 8 and -10.

We would be left over with -2.

$8 + -10 = -2$

Example:

$-3 + -6$

Here we would have no zero pairs as both numbers are negative.

$-3 + -6 = -9$

Subtracting Directed Number

$7 - 5$ is the same as $7 + -5$

$9 - 6$ is the same as $9 + -6$

$3 - 8$ is the same as $3 + -8$

$2 - -1$ is the same as $2 + 1$

$-4 - 3$ is the same as $-4 + -3$

Subtracting is the same as adding the inverse.

As with adding directed number, the counters help us as a representation of the subtraction.

We need to grow to not use the tiles but still use the ideas they represent.

Example:

$9 - -10$

Subtracting is the same as adding the inverse.

$9 + 10$

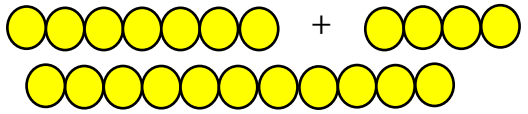
No zero pairs so group together.

$9 - -10 = 9 + 10 = 19$

Example:

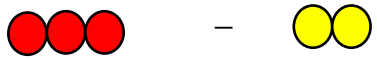
$7 - -4$ 

Subtracting is the same as adding the inverse.

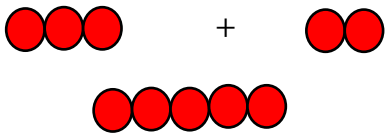


$7 - -4 = 11$

Example:

$-3 - 2$ 

Subtracting is the same as adding the inverse.



$-3 - 2 = -5$

Example:

$-15 - -4$

Subtracting is the same as adding the inverse.

$-15 + 4$

There will be 4 zero pairs here.

$-15 - -4 = -15 + 4 = -11$

Year 10 – Maths – Unit NP6 – Directed Number

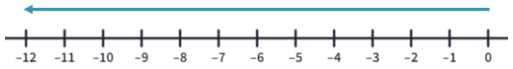
Multiplying Negative Numbers

In NP3 we thought about multiplication as a stretch of a vector We can apply this to negative numbers.

Example: -3×4



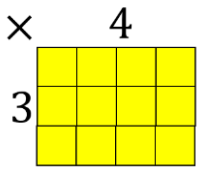
-3 stretched by a scale factor of 4.



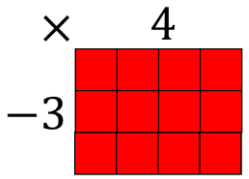
We also thought about multiplication as an area model. We will focus on this one now.

For -3×4

Set up 3×4 with tiles first



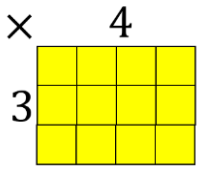
As there is one negative number, we flip the tiles once.

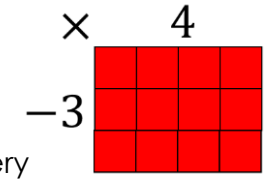


$$-3 \times 4 = -12$$

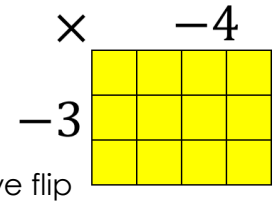
Now lets look at -3×-4

Set up 3×4 with tiles first





For every negative we flip the tiles once. So we flip for -3



Now we flip again for -4

Therefore, $-3 \times -4 = 12$

Just like with adding and subtracting, we use the tiles to help us represent it, but we want to grow to calculate it without the tiles.

Example:

$$-9 \times 12$$

Think of 9×12

$$9 \times 12 = 108$$

There is one negative number, so we 'flip' once.

$$-9 \times 12 = -108$$

Example:

$$-8 \times -7$$

Think of 8×7

$$8 \times 7 = 56$$

There is two negative numbers, so we 'flip' twice.

$$-8 \times 7 = -56$$

$$-8 \times= 7 = 56$$

Dividing Negative Numbers

We can apply the exact same way of thinking with dividing negative numbers (as dividing is the inverse of multiplying).

Example:

$$45 \div -9$$

Think of $45 \div 9$

$$45 \div 9 = 5$$

There is one negative number, so we 'flip' once.

$$45 \div -9 = -5$$

Example:

$$-18 \div -3$$

Think of $18 \div 3$

$$18 \div 3 = 6$$

There is two negative numbers, so we 'flip' twice.

$$-18 \div 3 = -6$$

$$-18 \div= 3 = 6$$

Year 10 – Maths – Unit NP7 – Fractions

My mathematical journey

What do I need to remember from before?

- Addition and subtraction (NP2)
- Multiplication and division (NP3)
- Exponents and roots (NP4)
- Order of operations (NP5)
- Directed numbers (NP6)

What will I learn about in this unit?

- Representing fractions with pictures and numerals
- Calculating with fractions
- Finding fractions and wholes

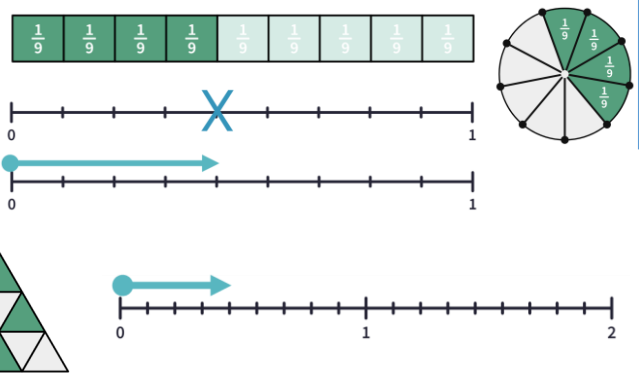
Where does this lead?

- Percentages, decimals and fractions (NP8)
- Proportional reasoning (NP10)
- Ratio (NP11)
- Linear equations (A4)
- Algebraic fractions (A17)

Representations of Fractions

We can represent the same fraction in different ways.

$\frac{4}{9}$



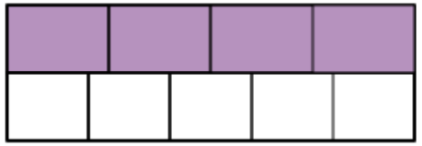
What is a Fraction?

numerator → $\frac{4}{9}$ ← We have 4 out of the 9 equal parts

fraction bar (vinculum) →

denominator → $\frac{4}{9}$ ← Each whole one is split into 9 equal parts

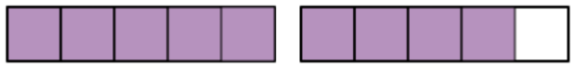
The diagram below does not represent $\frac{4}{9}$ because the parts are not all equal.



Improper Fractions & Mixed Numbers

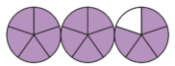
Improper Fractions

Each whole one is split into 5 equal parts. → $\frac{9}{5}$ ← We have 9 of those equal parts.



Example:

Convert $2\frac{4}{5}$ into an improper fraction.



We have 2 full whole ones. Each whole one is split into 5 equal parts.

$$2\frac{4}{5} = \frac{5}{5} + \frac{5}{5} + \frac{4}{5} = \frac{14}{5}$$

Mixed Numbers

We have 1 full whole one. → $1\frac{4}{5}$ ← We have another 4 of those equal parts.

Each whole one is split into 5 equal parts. →

Example:

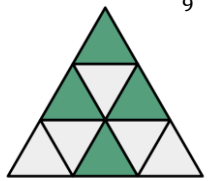
Convert $\frac{19}{5}$ into a mixed number.

Every $\frac{5}{5}$ is a whole one. We need to see how many groups of $\frac{5}{5}$ there are, and how many are left.

$$\frac{19}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{4}{5} = 3\frac{4}{5}$$

Complement of a Fraction

Shaded = $\frac{4}{9}$



Unshaded = $\frac{5}{9}$

The **complement** is the fraction needed to sum to 1.

Example:

Which symbol goes in the box < = >?

$$\frac{19}{3} \square 7\frac{1}{3}$$

Numbers need to be in the same type to compare.

$$\frac{19}{3} = 6\frac{1}{3}$$

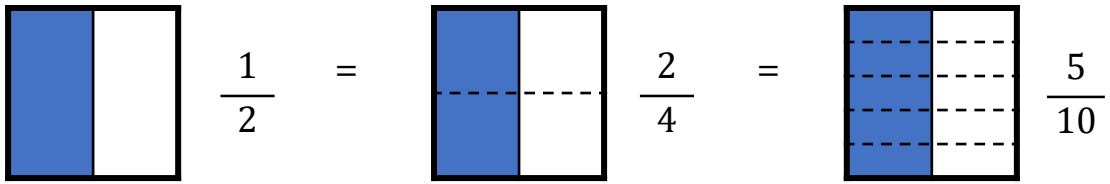
$$6\frac{1}{3} < 7\frac{1}{3}$$

Keyword/Skill	Definition/Tips
Equal parts	Fractions can only be described using equal parts of whole shapes.
Numerator	How many equal parts of a whole. The top number in a fraction.
Denominator	How many equal parts the whole is split into. The bottom number in a fraction.
Improper Fraction	A fraction where the **numerator** is bigger than the **denominator**.
Mixed Number	A number made from an integer and a fraction.
Equivalent Fractions	Two or more fractions that are equal in value.
Simplify	To cancel down a fraction to its lowest terms by dividing by all common factors.
Common Denominator	When two or more fractions have the same denominator.
Ascending Order	Order the numbers from the least value up to the greatest value.
Descending Order	Order the numbers from the greatest value to the least value.

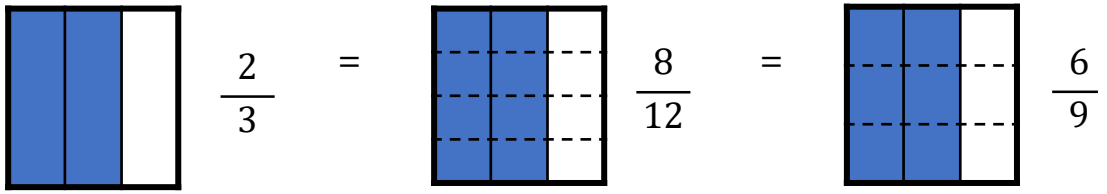
Year 10 – Maths – Unit NP7 – Fractions

Equivalent Fractions

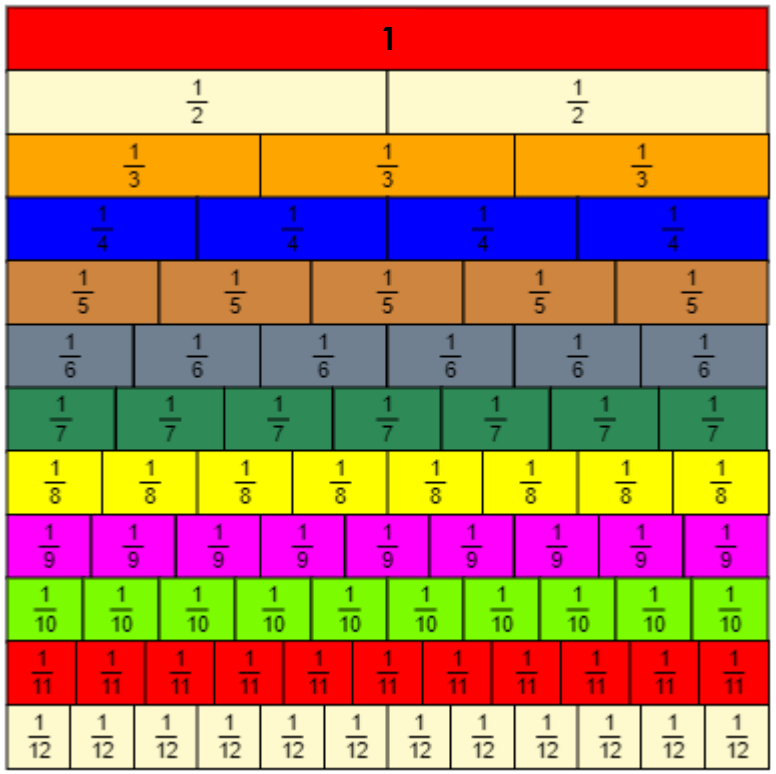
We can see that these fractions are **equivalent**.



$\frac{1}{2} = \frac{2}{4} = \frac{5}{10}$



$\frac{2}{3} = \frac{8}{12} = \frac{6}{9}$



A fraction wall is a useful way of finding fractions that are **equivalent**.

We can see that

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$$

because they are the same size on the fraction wall.

Can you spot any patterns in the numbers that also shows why they are equivalent?

Sneaky One

It is also helpful to recognise fractions equal to 1.



$\frac{4}{5} < 1$



$\frac{5}{5} = 1$



$\frac{6}{5} > 1$

'Sneaky One'

A fraction is equal to 1 if the numerator and denominator have the same value.

More examples: $\frac{10}{10}$ $\frac{3}{3}$ $\frac{120}{120}$

Using this can help us find equivalent fractions.

$\frac{1}{2} \times \frac{5}{5} = \frac{5}{10}$



$\frac{1}{2} = \frac{5}{10}$

Here are more examples of fractions equivalent to $\frac{1}{2}$

$\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$

$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$

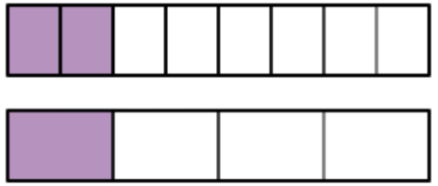
$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$

Simplifying Fractions

We can also divide by a 'sneaky one' without changing the value of the fraction. We call this simplifying.

$\frac{2}{8} \div \frac{2}{2} = \frac{1}{4}$

$\frac{2}{8} = \frac{1}{4}$



$\frac{1}{4}$ is the simplest form of $\frac{2}{8}$

You can always simplify in steps, but to simplify fully your 'sneaky one' needs to be the highest common factor of the numerator and denominator.

Example:

$$\frac{12}{18} \div \frac{2}{2} = \frac{6}{9} \div \frac{3}{3} = \frac{2}{3}$$

Using the HCF, we can do this in one step

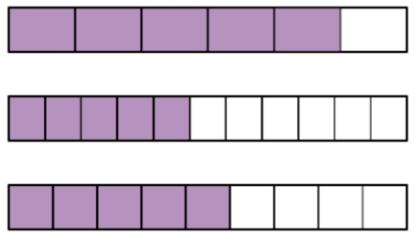
$$\frac{12}{18} \div \frac{6}{6} = \frac{2}{3}$$

Year 10 – Maths – Unit NP7 – Fractions

Ordering Fractions

To compare fractions there is a few things that can happen.
If the numerators are the same, we can compare the denominators.

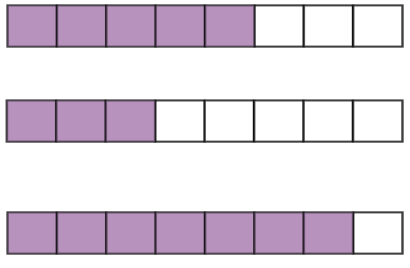
$\frac{5}{6}$
$\frac{5}{11}$
$\frac{5}{9}$



$$\frac{5}{11} < \frac{5}{9} < \frac{5}{6}$$

We can compare the numerators, if the denominators are the same.

$\frac{5}{8}$
$\frac{3}{8}$
$\frac{7}{8}$



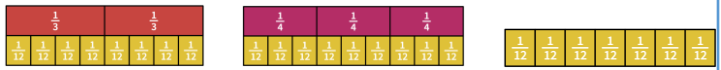
$$\frac{3}{8} < \frac{5}{8} < \frac{7}{8}$$

We can use equivalent fractions to help us compare sizes. This is the method you will have to use the most.

$$\frac{2}{3} \quad \frac{3}{4} \quad \frac{7}{12}$$

We can write $\frac{2}{3}$ and $\frac{3}{4}$ as twelfths (as 12 is the lowest common multiple of 3, 4 and 12. You can use any common multiple though).

$$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12} \qquad \frac{3}{4} \times \frac{3}{3} = \frac{9}{12} \qquad \frac{7}{12}$$

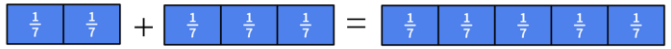


Now we can see that $\frac{7}{12} < \frac{2}{3} < \frac{3}{4}$

Adding & Subtracting Fractions

We can add fractions of the same type together by adding the numerators.

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$



We can subtract fractions of the same type by subtracting the numerators.

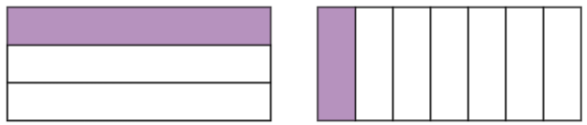
$$\frac{3}{7} - \frac{1}{7} = \frac{2}{7}$$



If the denominators are different, we need to find a common denominator before adding or subtracting.

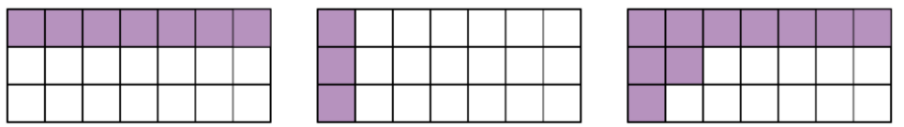
Example:

$\frac{1}{3} + \frac{1}{7}$



21 is the lowest common denominator here, as 21 is the lowest common multiple.

$\frac{7}{21} + \frac{3}{21} = \frac{10}{21}$



$$\frac{1}{3} + \frac{1}{7} = \frac{10}{21}$$

This method is the exact same for subtracting fractions (except you subtract the numerators, once you have a common denominator.

Example:

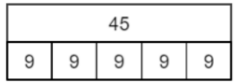
Year 10 – Maths – Unit NP7 – Fractions

Fraction of an Amount

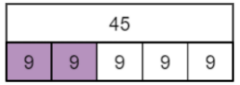
Let's find $\frac{2}{5}$ of 45

This means split 45 into five equal parts then find two of those parts

$\frac{1}{5}$ of 45 is 9



$\frac{2}{5}$ of 45 are 18



$\frac{2}{5}$ of 45 is found by $45 \div 5 \times 2 = 18$

split into five equal parts

find two of those parts

We can also find fractions of amounts when the fraction is greater than 1.

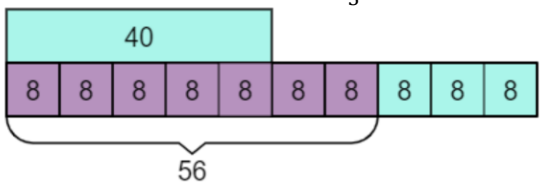
Example:

Calculate $\frac{7}{5}$ of 40

One whole is worth 40

$\frac{1}{5}$ of 40 is 8

$\frac{7}{5}$ of 40 = 56

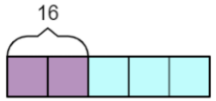


$\frac{7}{5}$ of 40 are 56

We can also find the whole amount of a number, given a fraction of it.

Example:

If $\frac{2}{5}$ of a number is 16, what is $\frac{1}{5}$ of the number?



If we know $\frac{2}{5}$ of the mount is 16, we can find one amount by dividing by 2.

$\frac{1}{5}$ of the amount will be $16 \div 2 = 8$

We now know 1 part of the 5 parts is 18.

Therefore, 5 parts (the whole amount) will be $5 \times 8 = 40$

40 is the whole amount!

Example:

If $\frac{3}{4}$ of an amount is £36, what is the full amount.

Divide by 3 to get one part: $36 \div 3 = 12$

Multiply by 4 to get the whole amount $12 \times 4 = 48$

The whole amount is £48.

Multiplying a Fraction by an Integer

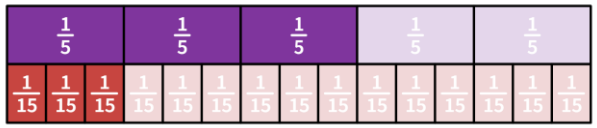
If we look at $\frac{1}{5} \times 3$. We can think of multiplication as 'lots of', so we have 3 lots of $\frac{1}{5}$.



$$3 \times \frac{1}{5} = \frac{3}{5}$$

We can see by this model here that we only need to multiply the numerator by the integer.

The model below shows why we don't multiply the denominator as well.



If we multiply the denominator as well, the value doesn't change.

Example:

$$\frac{1}{7} \times 3$$

$$3 \; lots \; of \; \frac{1}{7}$$

$$\frac{1}{7} \times 3 = \frac{3}{7}$$

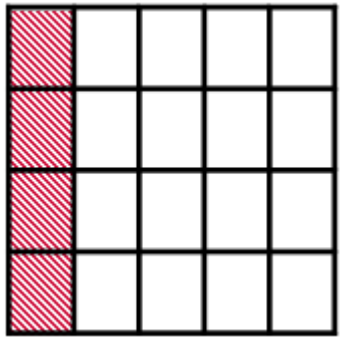
Year 10 – Maths – Unit NP7 – Fractions

Multiplying Fractions

When multiplying fractions, we can use a model to represent what happens.

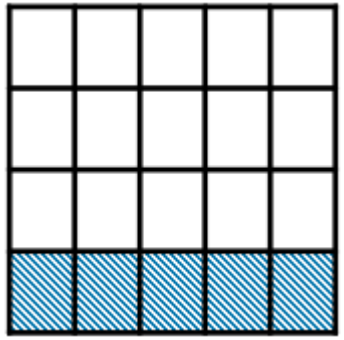
Example: We want to model $\frac{1}{4} \times \frac{1}{5}$

The red rectangle is $\frac{1}{5}$.



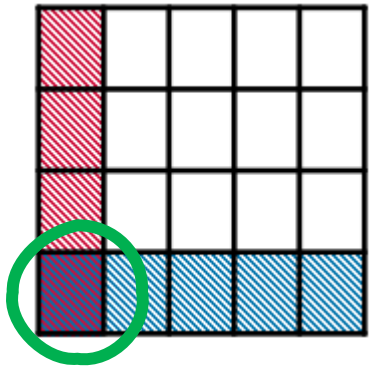
$\times$

The blue rectangle is $\frac{1}{4}$.



$=$

We can then combine the two diagrams



The crossover section is the answer.

It represents $\frac{1}{4}$ of $\frac{1}{5}$ **or** $\frac{1}{5}$ of $\frac{1}{4}$

Therefore, $\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

This shows that when we multiply fractions, we multiply the numerators together and multiply the denominators together.

Example:

$$\frac{2}{7} \times \frac{3}{4}$$

$$\frac{2 \times 3}{7 \times 4} = \frac{6}{28}$$

Always check if you can simplify your answer.

$$\frac{6}{28} \div \frac{2}{2} = \frac{6 \div 2}{28 \div 2} = \frac{3}{14}$$

$$\frac{2}{7} \times \frac{3}{4} = \frac{3}{14}$$

Multiplying Mixed Numbers

Before we multiply with mixed numbers. We need to convert the mixed numbers into improper fractions.

Example:

$2\frac{2}{5} \times 3\frac{1}{4}$ We need to convert both numbers into improper fractions. $2\frac{2}{5} = \frac{12}{5}$ and $3\frac{1}{4} = \frac{13}{4}$

Once they are improper fractions we can use the same method.

$$\frac{12}{5} \times \frac{13}{4} = \frac{12 \times 13}{5 \times 4} = \frac{156}{20}$$

As usual, always check if your answer can simplify. You can do this in steps if you need to.

$$\frac{156}{20} \div \frac{2}{2} = \frac{78}{10} \div \frac{2}{2} = \frac{39}{5}$$

It will usually ask you to convert your answer back into a mixed number.

How many $\frac{5}{5}$ can we get out of $\frac{39}{5}$: 7 with 4 left over.

$$\frac{39}{5} = 7\frac{4}{5}$$

Year 10 – Maths – Unit NP7 – Fractions

Reciprocal

The reciprocal of a number is the multiplicative inverse: the number you multiply it by to make 1.
A number and its reciprocal have a product of 1.

$$\frac{1}{2} \times 2 = \frac{2}{2} = 1$$

2 is the reciprocal of $\frac{1}{2}$

$\frac{1}{2}$ is the reciprocal of 2.

Another example: $\frac{3}{5}$ is the reciprocal of $\frac{5}{3}$

Dividing a Fraction by an Integer

$$\frac{1}{5} \div 3$$

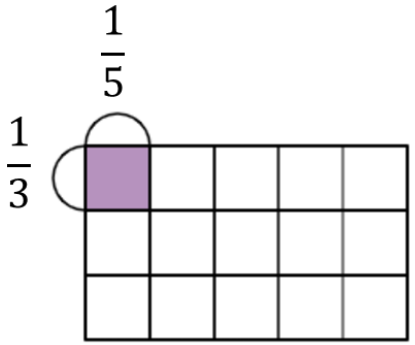
We can take $\frac{1}{5}$ and split it into 3 equal parts.



We can see that the answers is $\frac{1}{15}$

This is the same as $\frac{1}{5} \times \frac{1}{3}$

Dividing by 3 is the same as multiplying by $\frac{1}{3}$



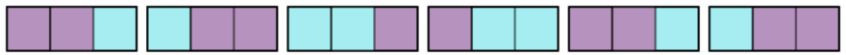
Dividing by a number or a fraction, is the same as multiplying by the reciprocal

Dividing an Integer by a Fraction

$$6 \div \frac{2}{3}$$



We can see how many $\frac{2}{3}$ go into 6.
We can show this by splitting it into thirds and then alternating the colours.



There are 9 groups of $\frac{2}{3}$, so $6 \div \frac{2}{3} = 9$

You can always multiply by the reciprocal for this too.

$$6 \div \frac{2}{3}$$

Reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

$$6 \times \frac{3}{2} = \frac{18}{2} = 9$$

Dividing Fractions

$$\frac{8}{9} \div \frac{2}{3}$$

We always come back to **dividing by a fraction is the same as multiplying by the reciprocal.**

$$\frac{8}{9} \times \frac{3}{2} = \frac{24}{18}$$

$$\frac{24 \div 6}{18 \div 6} = \frac{4}{3} = 1\frac{1}{3}$$

Same with multiplying mixed numbers, convert them into improper fractions before dividing.
Example:

$$2\frac{1}{3} \div 1\frac{1}{2}$$

$2\frac{1}{2} = \frac{7}{3}$ and $1\frac{1}{2} = \frac{3}{2}$

$$\frac{7}{3} \div \frac{3}{2} = \frac{7}{3} \times \frac{2}{3} = \frac{14}{9}$$

$$\frac{14}{9} = 1\frac{5}{9}$$